Mandatory vs. voluntary disclosure in the dynamic market for lemons[*]

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Abstract

We consider a dynamic adverse selection setting where a privately informed seller can choose to reveal or withhold past trade information to privately informed buyers. Buyers naturally receive less information when the seller can strategically withhold negative news relative to a setting where current buyers always observe the seller's history of trade, i.e., mandatory disclosure. Despite the informational disadvantage, we find that strategic disclosure by the seller can be *welfare-increasing* relative to mandatory disclosure, under which past trade is always disclosed. This occurs because voluntary disclosure attenuates the seller's incentive to engage in destructive signaling and can lead to more efficient trade.

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JEL classification: C72, D82, D83, G11, G23

1   Introduction

In a number of asset or service markets, prior trade information is typically not readily observable by interested buyers. Indeed, few markets mandate that such information be disclosed to current potential buyers. For example, in the used car industry, a buyer who arrives at a dealership typically cannot readily observe whether the dealer has sold or failed to sell cars to previous buyers. Likewise, when hiring a contractor to undertake a project, such as construction or the provision of services, the lack of interest in a given contractor by past buyers is generally not observable. Similarly, employers may not easily discover a potential worker's full employment history. The past sale of illiquid securities in most over-the-counter markets relatedly does not need to be publicly disclosed. Consequently, buyers typically cannot observe the full history of sales, or the lack thereof, for a given seller.

However, a used car dealer or contractor who *has* engaged in past trade can voluntarily disclose such information to present buyers, and can do so credibly since such information is verifiable. Continuing with the previous examples, following the sale of a car, the dealer can present a copy of the bill of sale provided to the previous buyer, indicating the price at which trade took place. Indeed, such a practice is common, with dealers typically announcing recent sales of used vehicles and the price at which they were sold. The contractor or securities dealer can likewise furnish prior deals and their agreed prices to newly interested buyers; the worker can similarly produce documents of past employment. The absence of trade affords no such luxury for the seller—it is difficult to prove a negative that a sale did *not* take place. At the same time, the seller can strategically hide past trade information from buyers, if, for example, such trade reflects poorly on the seller.

Related to this discussion, transparency of past trade and prices in various markets has become increasingly prominent in public policy debates. Indeed, the conventional wisdom has been that greater transparency and a stronger information environment are generally better for market participants. For example, in an effort to "level the playing field," regulators mandated the public disclosure of trades, including prices and volumes, for bonds and other fixed income facilities through the Transactions Reporting and Compliance Engine (TRACE), first implemented in 2002 for liquid bonds and gradually expanded to other bonds over four phases.[1] The empirical evidence on the effects of TRACE, however, is mixed. Bessembinder et al. (2006) find positive early evidence of lower transaction costs for bonds

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[1]Mandatory disclosure of trades under TRACE has since been expanded to other debt securities as well, including expansion to agency debt securities in 2010, asset-backed securities and mortgage-backed securities in 2011, U.S. Treasury securities in 2017, and most recently, U.S. dollar-denominated foreign sovereign debt securities in 2023.

exposed to TRACE in the first phase. Conversely, when considering all phases, Asquith et al. (2019) document an average 11.7% decrease in the number of trades, driven largely by trade reduction in illiquid bonds exposed in later phases of TRACE.

The European Union has likewise passed regulations requiring the disclosure of trade information in decentralized European bond markets, through the 2018 implementation of the Markets in Financial Instruments Directive (MiFID II). The U.S. municipal bond market experienced a similar shift towards transparency of trades between 1995–2005. The SEC has likewise taken steps to increase transparency of asset sales in other markets, such as securities-based swaps.[2] Similarly, in the healthcare industry, regulators imposed that insurers, who engage in bargaining with hospitals for the prices of their services, must publicly disclose these negotiated prices, implemented in July 2022 (Kona and Corlette, 2022). These examples illustrate that the issue of transparency in trade is highly relevant (and highly debated) among policymakers and practitioners.[3]

We seek to study the phenomenon of voluntary disclosure of past trade in a dynamic adverse selection model, as well as examine the efficiency and welfare implications of voluntary disclosure relative to a setting where buyers always observe past trade, i.e., mandatory disclosure. The questions we seek to address are thus: (i) What are the economic forces that determine trade and disclosure, or the lack thereof, in repeated bargaining? and (ii) How does the presence of voluntary disclosure compare to situations under which buyers always observe past trade information? Given the interest in mandatory disclosure among policymakers and regulators, it is important to better understand the role of voluntary disclosure in asset markets and the welfare and trading volume implications relative to mandatory disclosure.

Our setting is one where a seller can trade one unit of an indivisible asset in each of two periods to short-lived buyers. The seller is privately informed about the asset's quality (which determines the seller's and buyers' values), while buyers receive imperfect private signals regarding quality. Buyers in each period simultaneously make private offers to the seller, who can decide to accept or reject such offers. The seller has discretion over the trade information that buyers observe. Specifically, following trade of the asset in the first period, the seller can disclose to second-period buyers that trade took place in the first period and the trading price. The seller can also choose to withhold such information from buyers.

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[2]See, for example, "Statement on Public Dissemination of Security-Based Swap Transactions," by the Securities and Exchange Commission, February 16, 2022.

[3]Regarding the debate around transparency regulation, see "SEC Hit With Challenge Over One-Minute Trade Reporting Plan," Bloomberg Law News, November 18, 2024, and "MSRB Rescinds One-Minute Trade Reporting Standard, Responding to Industry Feedback and Concerns," MSRB press release, June 9, 2025.

However, in the absence of trade, the seller cannot credibly disclose that trade did not occur. As noted in the examples above, disclosure of trade is verifiable—records are produced during the transaction that can verify to second-period buyers that trade occurred. Conversely, the absence of trade is more difficult to verifiably prove, as such a strategy can be mimicked by sellers who have traded. We refer to this information structure as the *voluntary disclosure regime*.

We find that, under the voluntary regime, sellers disclose early trade that occurred at a high price to future buyers. This signals to later buyers that at least one early buyer privately observed positive information, which positively influences buyer beliefs in the second period. In contrast, if trade occurred at a low price, then the seller chooses to strategically withhold such information from future buyers. At the same time, a seller who has rejected all first-period offers is limited to non-disclosure, as communication of no trade occurring is not verifiable. Consequently, second-period buyers are uncertain as to whether non-disclosure is due to a high-type seller who has rejected all offers or because a low-type seller has accepted a low offer. While the former is positive information, which can improve buyer beliefs, the latter is negative information. As a result of this pooling behavior by the low-type seller, we find that second-period buyers revise beliefs downward following non-disclosure.

We compare the voluntary regime to a setting where sellers are required to disclose any trade that occurred along with its price in the first period, thereby allowing second-period buyers to observe the trade history. We refer to this setting as the *mandatory disclosure regime*. Under mandatory disclosure, the low-type seller can no longer conceal that trade at a low price had taken place, allowing the high-type seller to use rejection of first-period offers to signal her type. It would therefore appear that the voluntary disclosure regime should be efficiency-dominated by the mandatory regime. However, we find that the opposite can be true under certain conditions. Indeed, in our main result, we show that, despite the apparent informational disadvantage, *strategic disclosure by the seller can be welfare-improving relative to mandatory disclosure* (Theorem 1).

The reason for this is two-fold. Under mandatory disclosure, rejection of early offers by the seller conveys to second-period buyers that asset value is more likely to be high. However, the low-type seller has an incentive to mimic the high-type seller's strategy of early rejection, resulting in loss of trade and welfare. This incentive can be so strong that the low-type seller rejects low offers with a high probability in equilibrium, including with probability one. The strong mimicry incentives of the low-type seller in this situation are welfare destroying: there is a low likelihood of trade not only in the first period, but also in the second period, as

second-period buyers update less following first-period rejection.

In contrast, under voluntary disclosure, the seller is able to hide low-price trades while revealing high-price ones. Moreover, the seller is unable to credibly communicate that she has rejected offers. As a result, the low-type seller always engages in profitable trade in the first period. In other words, voluntary disclosure attenuates the seller's incentive to engage in welfare-reducing mimicry; indeed, we find that destructive signaling occurs only when all information is mandatorily disclosed. Voluntary disclosure thus represents a less costly communication channel for the seller relative to signaling through rejection. Despite the seller suppressing negative information in the voluntary regime (and total information often being higher under mandatory disclosure), heightened trade in the voluntary regime improves welfare, leading this regime to be welfare-enhancing relative to the mandatory regime when the low-type seller's mimicry incentive is strong.

We similarly find that trading frequency (or volume) is greater under the voluntary regime compared to mandatory disclosure when the signaling incentive is high. An interesting feature of this analysis is that more information is generally considered to be desirable in trade. However, our results suggest that additional information for buyers can be *Pareto destructive* and reduce welfare. This makes voluntary disclosure appealing as it can undo the destructive mimicry incentives under mandatory disclosure.

In additional analysis, we extend our baseline two-period setting to a model with arbitrarily many periods. In this analysis, we show that the main economic insights of our baseline setting continue to hold in a longer horizon. Specifically, we find that, when the discount factor is sufficiently low, voluntary disclosure can be welfare-enhancing relative to mandatory disclosure. This is due to the fact that the efficiency gains from voluntary disclosure can outweigh gains from more efficient trade in the long-run under the mandatory regime. This result is perhaps surprising, as a lower discount factor weakens the low-type seller's signaling incentive, thus weakening the edge of voluntary over mandatory disclosure. However, we find that the signaling costs saved under voluntary disclosure are a first-order driver of the welfare benefits.

Furthermore, we similarly examine welfare under voluntary and mandatory disclosure relative to an opaque regime where the seller is unable to disclose trade information. We find that voluntary disclosure welfare-dominates the opaque regime as well, under similar conditions where voluntary dominates mandatory disclosure. Additionally, in extensions, we explore a benchmark setting where disclosure only entails that trade occurred and not the trading price, the baseline model with an imperfectly persistent state, and stochastic buyer

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arrival.

Our model has implications for transparency regulation, particularly in OTC markets, a long-standing debate in public policy (Bessembinder and Maxwell (2008), Asquith et al. (2019)). As noted previously, the introduction of TRACE in the decentralized OTC bond market aimed to facilitate trade by allowing investors to observe the prices and volumes of prior trades. In our setting, we can interpret the seller's reservation payoff following no trade in a period as the inverse inventory cost of holding the bond for that period (e.g., a lower inventory cost corresponds to a higher reservation payoff). The results of the model imply that, when adverse selection frictions are not too severe, voluntary disclosure is advantageous relative to both mandatory disclosure and an opaque regime for bonds with low enough or high enough inventory costs. This suggests that policymakers can improve upon the current transparency framework by permitting voluntary disclosure of trades for certain classes of bonds. We discuss implications and the related empirical literature further in Section 3.5.

1.1   Related literature

Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) first study voluntary disclosure and establish that, in the absence of disclosure frictions, the sender always reveals her private information, often referred to as the unraveling result. Voluntary disclosure has since been studied in repeated and dynamic settings. Einhorn and Ziv (2008) examine a repeated setting where a manager who withholds disclosure can build a reputation for being uninformed, thereby saving future disclosure costs. Beyer and Dye (2012) likewise consider a repeated game where the manager can be either forthcoming or strategic. They find that a strategic manager will disclose bad news to mimic the forthcoming type. In dynamic settings, Dye and Sridhar (1995), Shin (2003), Acharya et al. (2011), Guttman et al. (2014), Ben-Porath et al. (2018), Aghamolla and An (2021), and Bertomeu et al. (2022) examine voluntary disclosure when the sender has intertemporal considerations. Our setting differs from these studies principally in that we consider voluntary disclosure in the context of trade with strategic counter-parties, whereas trade is absent in the above-mentioned papers.

A recent literature examines strategic disclosure by an informed party to an uninformed party in bargaining. Glode et al. (2018) consider disclosure via Bayesian persuasion in single-period bilateral bargaining. Li et al. (2022) study an auction for takeovers where an informed buyer can disclose hard information to an uninformed competitor. In these settings, agents can strategically disclose information about the asset value specifically. In contrast, the seller is unable to disclose asset value in our setting, but can disclose past trade information.

Moreover, both parties have private information in our setting, while these studies feature an uninformed counter-party. Finally, our model is dynamic, while these papers examine single-period bargaining.

Our study also relates to the literature on the effects of the mandatory dissemination of information in financial markets (see Goldstein and Yang (2017) for a review). The conventional wisdom is that greater public information disclosure has positive market effects, such as improved liquidity and efficiency and lower cost of capital and volatility (e.g., Diamond and Verrecchia (1991), Easley and O'hara (2004), Hughes et al. (2007), Lambert et al. (2007)). However, a number of studies examining welfare implications find negative welfare effects of public information. Hirshleifer (1971) shows that more disclosure can harm welfare by reducing risk-sharing trades. Relatedly, Kurlat and Veldkamp (2015) find that increased disclosure by firms can harm investor welfare by reducing access to assets with higher risk and return. In terms of efficiency, Goldstein and Yang (2019) examine a trading setting where managers can learn from prices and find that increased disclosure can crowd out managerial learning, thereby reducing investment efficiency. In coordination settings, Morris and Shin (2002), Angeletos and Pavan (2004), Goldstein et al. (2011), Colombo et al. (2014), and Vives (2017) examine the welfare effects from public information on coordination motives when agents can have (or can acquire) private information. The above-mentioned studies generally examine mandatory public disclosure relative to no information or by varying the precision of a public signal. Our study contributes to the broad literature on the role of information in financial markets by identifying that *voluntary* disclosure—where information is strategically released by an interested party—can mitigate the negative effects of mandatory information dissemination and therefore be welfare-enhancing relative to both mandatory disclosure and no information disclosure.

This study also contributes to the recent literature on dynamic adverse selection, such as Hörner and Vieille (2009), Daley and Green (2012, 2016), Kaya and Liu (2015), Fuchs et al. (2016), and Asriyan et al. (2017, 2021). These studies all consider single-asset settings where the seller can only trade once, whereas our framework features repeated trading opportunities. A few studies in this literature examine the role of information of past trade. Kim (2017) considers a setting where buyers arrive stochastically and can only observe the amount of time the seller has been on the market, rather than the arrival of past offers. In repeated trade models, Kaya and Roy (2022) examine a setting where present buyers can observe a partial history of the seller's past trade behavior, while Kaya and Roy (2024) consider the welfare effects of buyer information of trade history in light of the number of

buyers in each period (monopsony versus competition). Our study differs as we allow the seller to control the trade information that buyers see, including the price at which trade took place.

Back et al. (2020) consider a setting where a dealer (intermediary) first purchases an asset and then must sell the asset on the secondary market. Transparency of the dealer's acquisition cost of the asset leads the dealer to overpay for the asset relative to his private information, in an attempt to gain a higher price in the secondary market. The result is perfect separation and the overpayment is undone by buyers in the secondary market, resulting in perfect revelation of the dealer's signal, similar to the signaling effects in signal-jamming models (e.g. Stein, 1989). Among other differences, we consider a dynamic model with repeated sales and reputation concerns, where the seller can only build reputation through past sales. Moreover, the seller in our model has a reservation payoff, thereby operationalizing the lemons problem, whereas trade following acquisition in Back et al. (2020) occurs with probability one by assumption. Finally, we analyze strategic disclosure, which is absent in Back et al. (2020).

Furthermore, as our setting includes buyer private information, our study relates to the small but growing stream of literature that embeds buyer private information in dynamic adverse selection models, such as Kaya and Kim (2018) and Aghamolla and Hashimoto (2022). Kaya and Kim (2018) consider stochastic buyer arrival, while Aghamolla and Hashimoto (2022) endogenize the time in which buyers can make offers to the seller. In contrast, we examine the interaction of buyer private information with strategic disclosure by the seller, which is absent in these settings.

The paper proceeds as follows. In the following section, we outline the model. In Section 3, we present the equilibrium of the voluntary and mandatory disclosure regimes and our main results concerning efficiency comparisons. In Section 4, we explore the model in a long horizon, and the final section concludes. Additional analyses are included in Section A and all proofs relegated to Section B.

2 Model

We consider a long-lived seller who is endowed at the beginning of each period $t \in \{1, 2\}$ with an indivisible asset.[4] In each period, two short-lived buyers arrive and make simultaneous

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[4]In Section 4, we consider an arbitrarily long horizon.

offers.[5] The quality of the asset is determined by the state $\theta \in \{L, H\}$ and is persistent across periods. The prior probability of the high state, $\theta = H$, is denoted by $\pi_0 \in (0, 1)$. The seller's outside option or reservation value for an asset of quality θ is c_θ, where $c_H > c_L$, while buyers value the asset at v_θ, where $v_H > v_L$. We assume that there is common knowledge of gains from trade given the state, i.e., $v_\theta > c_\theta$.

The seller is privately and perfectly informed of the true state at the beginning of the game. Each buyer in each period receives a private and imperfect signal of θ, denoted by $s_i \in \{\ell, h\}$, that is correct with probability $q \in (1/2, 1)$, i.e.,

$$\mathbb{P}(s_i = h | \theta = H) = \mathbb{P}(s_i = \ell | \theta = L) = q.$$

Buyer private signals are independent conditional on the state. For ease of exposition, we often refer to a buyer who privately received signal $s_i \in \{\ell, h\}$ as an s_i-buyer and the seller of type $\theta \in \{L, H\}$ as a θ-seller.

After observing their private signals, each buyer simultaneously makes a private offer, denoted by $x_i \geq 0$, to the seller. In each period, the seller can either accept one offer or reject both offers. If the seller accepts an offer and trade occurs, the winning buyer in period t receives the payoff

$$u_b = v_\theta - p_t, \tag{1}$$

where p_t denotes the winning offer, or trade price, accepted by the seller in period t. Any buyer whose offer is rejected receives a payoff of zero.

The seller receives the payoff of p_t upon acceptance of an offer $x_i = p_t$ in period t.[6] In the event that trade does not take place in a given period, the seller receives a payoff equal to her reservation value or outside option, c_θ.[7] At the end of each period, the seller consumes her payoff for that period; regardless of the sales outcome in the previous period, the seller provides a single new asset in each period.

The seller discounts future payoffs by a factor of $\delta \in (0, 1]$. Let $z_t = 1$ denote the event

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[5]The assumption of short-lived buyers is widely used in the dynamic adverse selection literature; see, e.g., Daley and Green (2012, 2016), Kim (2017), Asriyan et al. (2017), among many others.

[6]We use the pronoun "she" when referring to the seller and "he" when referring to a buyer.

[7]We may alternatively assume, as in Kaya and Roy (2022, 2024), that c_θ represents the seller's cost of production for the asset and that the seller only incurs this cost if trade occurs, resulting in a payoff of zero in periods without trade. In the solution, this alternative assumption is mathematically equivalent to the assumption we use of c_θ as the seller's reservation payoff. Hence, all of our results are unchanged under this alternative assumption.

that trade occurs in period t and $z_t = 0$ for no trade. The seller's ex ante payoff is

$$
u_s = \begin{cases}
p_1 + \delta c_\theta, & \text{if } (z_1, z_2) = (1, 0), \\
c_\theta + \delta p_2, & \text{if } (z_1, z_2) = (0, 1), \\
p_1 + \delta p_2, & \text{if } (z_1, z_2) = (1, 1), \\
c_\theta + \delta c_\theta, & \text{if } (z_1, z_2) = (0, 0).
\end{cases}
\tag{2}
$$

At the end of the first period, both buyers exit and two new buyers arrive at the beginning of the second period. We consider the following two regimes for the second-period buyers' information about the seller's trading behavior in period 1.

- *Voluntary disclosure regime.* If trade occurred in the first period, the seller can choose to publicly disclose to buyers the trading price (which implies that trade took place) at the start of the second period. Likewise, the seller can choose to withhold this information and disclose nothing. In the event that trade did *not* occur in $t = 1$, the seller cannot credibly disclose this information. The reason for this difference is that disclosure is meant to be *verifiable.* For example, following trade, the seller can obtain documentation that a trade occurred and the trading price, such as a bill of sale, while she cannot prove the negative that no trade occurred. This feature is common to models of voluntary disclosure, where certain sender types are unable to communicate if their information is unverifiable (e.g., Dye, 1985; Acharya et al., 2011; Ben-Porath et al., 2018).

- *Mandatory disclosure or transparent regime.* Under mandatory disclosure, the seller is required to disclose the trading price at the start of the second period if trade occurred in $t = 1$. That is, the seller can no longer withhold trade information. In the event of no trade, the seller discloses nothing. Second-period buyers in this regime therefore always observe trade and the price if trade had occurred in the previous period.

In either regime, buyers do not observe the offers made in a prior period, unless the seller discloses the winning offer following trade.[8] Moreover, we include the following standard assumptions:

Assumption 1. $(1 + \delta)v_L < c_L + \delta v_H$.

Assumption 2. $v_L < c_H$.

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[8]In other words, rejected offers are never observed by future buyers.

The first assumption is only relevant in the mandatory regime. If Assumption 1 is violated, then the low-type seller would strictly prefer to trade in the first period with probability one in the mandatory regime, thereby revealing her type with certainty, resulting in perfect separation. We exclude this case to focus the analysis on the more interesting situation where the low-type seller has a mimicry incentive. The second assumption is the standard lemons condition. If Assumption 2 is violated, trade always occurs regardless of buyers' beliefs. The sequence of the stage game for a generic period t is summarized as follows:

Stage 1: The seller is endowed with an asset, with value indexed by the state θ. The seller perfectly observes the state.

Stage 2: Two buyers arrive. In any period $t > 1$, if trade occurred in period $t - 1$ and under the voluntary regime, the seller makes a decision to publicly disclose p_{t-1} or to keep quiet. Under the mandatory regime, the seller must disclose p_{t-1} if trade occurred in period $t - 1$.

Stage 3: The buyers observe imperfect private signals of the state θ.

Stage 4: Buyers simultaneously make one-time offers to the seller. The seller chooses to accept one offer or to reject both offers. Both buyers exit. The seller consumes her t-period payoff.

Solution concept. The equilibrium concept we employ is perfect Bayesian equilibrium. We consider equilibria in symmetric strategies, whereby all buyers of a certain type, ℓ or h, use the same strategy. Due to the presence of off-equilibrium-path beliefs in our setting, we employ refinements in the equilibrium analysis of both regimes to rule out non-credible equilibria supported by extreme off-path beliefs. In the mandatory regime, we use the D1 criterion developed by Cho and Kreps (1987), which is a standard refinement used in signaling models with costly actions (as we show later, rejection of low offers by the L-seller is costly in the mandatory regime). In contrast, in the voluntary regime, disclosure deviations are costless and thus the D1 criterion cannot be applied (Bertomeu and Cianciaruso, 2018). We therefore utilize the Grossman-Perry-Farrell equilibrium (GPFE) refinement of Grossman and Perry (1986) and Farrell (1993), which is often used in models with verifiable messages (e.g., Glode et al., 2018).[9] The GPFE refinement is closely related to the D1 criterion but is applicable to communication settings where actions do not bear a direct cost for the sender.

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[9] The GPFE criterion is also used in Gertner et al. (1988), Lutz (1989), Maskin and Tirole (1992), Severinov (2008), and Perez-Richet (2014), among others.

We provide further details in the equilibrium analysis below as well as in Appendix C.

3 Equilibrium

3.1 No-disclosure benchmark

We begin with a benchmark analysis in which the seller is unable to disclose any prior trade information to second-period buyers. The model therefore reduces to a static analysis in each of the two periods, with buyers relying only on their private signals to update on asset value and not the trade history. This analysis is helpful in understanding the role of disclosure in shaping buyer beliefs and offer strategies.

To characterize offer strategies, we consider separately optimistic and pessimistic beliefs. We find that the offer strategies of the buyers depend critically on the belief level prior to observing private signals. We begin with the case where the adverse selection problem is the most severe and then consider progressively more optimistic beliefs. Under severe adverse selection, buyer beliefs satisfy the lemons condition:

$$\mathbb{E}[v_\theta | s_i = h; \pi] \leq c_H. \tag{Case 1}$$

Under Case 1, a buyer's posterior expectation of the asset is below c_H even after observing the high signal and an initial belief π. In this case, neither buyer makes an offer that is acceptable to the H-seller and only the low-type seller is targeted. As such, Bertrand competition drives both offers to the zero-profit level v_L.

Naturally, as beliefs become more optimistic, buyers begin to target the high-type seller. Consider an improvement of beliefs from the lemons condition to a low-intermediate level:

$$\mathbb{E}[v_\theta | s_i = h; s_j = \ell; \pi] \leq c_H < \mathbb{E}[v_\theta | s_i = h; \pi]. \tag{Case 2}$$

In this case, a buyer who receives the high signal is optimistic enough to target the H-seller, but does not find it profitable to do so if it means winning against a low-signal buyer (i.e., the winner's curse). As such, the equilibrium offer strategy for an h-buyer entails winning against another h-buyer with some probability. In this way, h-buyers find it worthwhile to (probabilistically) target the high-type seller. Hence, the h-buyer's offer strategy must place a discrete probability on the offer v_L—the same offer that is made by a buyer who observes the low signal.

Next, because the other buyer may be offering v_L, an h-buyer can win against such a buyer by simply offering c_H, the lowest acceptable offer to the H-seller. However, such a strategy can be beaten by another offer that is, for example, $c_H + \varepsilon$. As such, the h-buyer mixes over offers that also target the H-seller according to an atomless distribution over support $[c_H, \mathbb{E}[v_\theta | s_i = h; \pi]]$. The reason that the h-buyer's offer when targeting the H-seller is not a single point is due to the presence of v_L offers, which leads to mixing in the offer distribution targeting the H-seller. In sum, ℓ-buyers offer v_L with probability one, while h-buyers mix between offering v_L with discrete probability and an offer (also randomized) that also targets the high-type seller over support $[c_H, \mathbb{E}[v_\theta | s_i = h; \pi]]$.

Similar offer strategies emerge as beliefs become more optimistic. We next consider the following high-intermediate belief level:

$$\mathbb{E}[v_\theta | s_i = \ell; s_j = \ell; \pi] < c_H < \mathbb{E}[v_\theta | s_i = h; s_j = \ell; \pi]. \qquad \text{(Case 3)}$$

In this case, h-buyers always target the H-seller, while ℓ-buyers continue to only target the low-type seller with offer v_L. As in the previous case, because there is a positive probability that the other buyer received signal ℓ, h-buyers can always win against this buyer type and therefore h-buyers continue to mix over offers that target the high-type seller. An interesting property is that ℓ-buyers do not target the H-seller here, even though we can have $\mathbb{E}[v_\theta | s_i = \ell; \pi] > c_H$ in this case. Because ℓ-buyers always have a lower posterior expectation than buyers who received signal h, an ℓ-buyer that targets the H-seller and wins is likely winning against another ℓ-buyer, resulting in strictly negative profit. As such, ℓ-buyers do not target the H-seller, and Bertrand competition drives their offers to the zero-profit level v_L.

Finally, when beliefs are the most optimistic, i.e.,

$$\mathbb{E}[v_\theta | s_i = \ell; s_j = \ell; \pi] > c_H, \qquad \text{(Case 4)}$$

buyers of either type make offers acceptable to the H-seller. Buyers who observe the low signal offer their posterior expectation due to Bertrand competition, while h-buyers mix over offers. Since posterior beliefs are always above c_H in Case 4, trade always transpires, and hence there is no adverse selection in this case.

These observations are summarized in the following lemma, which establishes equilibrium offer strategies for a given prior π.

Lemma 1. *Let $\pi \in [0, 1]$ be buyer beliefs before receiving private signals. If π is 0 or 1, the*

equilibrium offer is v_L or v_H, respectively. Otherwise, equilibrium offers are characterized as follows.

Case 1. *(Low beliefs.) $\mathbb{E}[v_\theta | s_i = h; \pi] \le c_H$. Buyers of either type offer v_L with probability one.*

Case 2. *(Low-intermediate beliefs.) $\mathbb{E}[v_\theta | s_i = h, s_j = \ell; \pi] \le c_H < \mathbb{E}[v_\theta | s_i = h; \pi]$. The ℓ-buyer offers v_L with probability one, while the h-buyer places positive mass σ on v_L and randomizes over $[c_H, \mathbb{E}[v_\theta | s_i = h; \pi]]$. That is, the h-buyer's offer x follows the distribution G_2 defined by*

$$
G_2(x) = \begin{cases} 0 & \text{for } x < v_L \\ \sigma & \text{for } x \in [v_L, c_H), \\ \widetilde{G}_2(x) & \text{for } x \in [c_H, \mathbb{E}[v_\theta | s_i = h; \pi]], \end{cases}
$$

where σ and the atomless function $\widetilde{G}_2$ are characterized in the Appendix.

Case 3. *(High-intermediate beliefs.) $\mathbb{E}[v_\theta | s_i = \ell, s_j = \ell; \pi] < c_H < \mathbb{E}[v_\theta | s_i = h, s_j = \ell; \pi]$. The ℓ-buyer offers v_L with probability one, while the h-buyer targets the H-seller according to an atomless distribution G_3 with support $[c_H, \overline{x}_3]$, where*

$$
\overline{x}_3 := \mathbb{P}(s_i = \ell | s_j = h) c_H + \mathbb{P}(s_i = h | s_j = h) \mathbb{E}[v_\theta | s_i = h, s_j = h; \pi].
$$

The cumulative distribution G_3 is characterized in the Appendix.

Case 4. *(High beliefs.) $\mathbb{E}[v_\theta | s_i = \ell, s_j = \ell; \pi] > c_H$. The ℓ-buyer offers $\mathbb{E}[v_\theta | s_i = \ell; s_j = \ell; \pi]$ with probability one, while the h-buyer uses an atomless distribution $G_4(b)$ with support $[\mathbb{E}[v_\theta | s_i = \ell, s_j = \ell; \pi], \overline{x}_4]$, where*

$$
\overline{x}_4 := \mathbb{P}(s_i = \ell | s_j = h) \mathbb{E}[v_\theta | s_i = \ell, s_j = \ell; \pi] + \mathbb{P}(s_i = h | s_j = h) \mathbb{E}[v_\theta | s_i = h, s_j = h; \pi].
$$

The cumulative distribution G_4 is characterized in the Appendix.[10]

Furthermore, the θ-seller accepts the highest offer that is greater than or equal to c_θ.

Importantly, we observe in Lemma 1 that trade does not occur with positive probability in Cases 1–3 where adverse selection is present, which is due to rejection of offers below c_H

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[10]In the knife-edge case, if $\mathbb{E}[v_\theta | s_i = \ell, s_j = \ell; \pi] = c_H$, which is the boundary of Cases 3 and 4, then the ℓ-buyer is indifferent between targeting the L-seller (offering v_L) and targeting the H-seller (offering c_H), and any randomization of these offers is part of the equilibrium. The h-buyer's strategy is the same as described in Cases 3 and 4.

(such as v_L) by the H-seller in these cases. This is due to the fact that offers of v_L (which are below c_H) occur with positive probability in Cases 1–3. Hence, the H-seller rejects offers even in the absence of signaling concerns in this no-disclosure benchmark. This property of rejection only by the H-seller gives rise to the mimicry incentive to reject offers by the L-seller under the mandatory regime.

3.2 Voluntary regime

We now analyze the equilibrium of the voluntary disclosure regime. Under voluntary disclosure, the seller only discloses the trading price of the first period, p_1 (if trade took place), if this disclosure improves buyer beliefs in the second period. Let $d \in \{D, ND\}$ denote the event of disclosure ($d = D$) or non-disclosure ($d = ND$). As noted above, if trade does not occur in the first period, the seller is unable to disclose. Moreover, a seller that has traded in the first period can choose to withhold disclosure, thus mimicking the disclosure behavior of a seller for which trade did not occur.

We first conjecture a disclosure strategy by the seller and later confirm that it indeed constitutes an equilibrium disclosure strategy (as well as uniqueness). We conjecture that the seller only discloses p_1, thus indicating that trade took place in period 1, for prices that would be acceptable to the H-seller. Because the seller's reservation payoff is c_θ in the event of no trade, it is natural to conjecture that the type-θ seller does not accept any offer below c_θ. Hence, we conjecture that disclosure only occurs for $p_1 \geq c_H$. Disclosure of $p_1 < c_H$ indicates that $\theta = L$ with probability one and thus period-2 buyers would only target the low-type seller following this disclosure.

The disclosure strategy above also determines the seller's acceptance strategy in period 1. Given that the L-seller can withhold negative information (i.e., trade where $p_1 < c_H$), this seller accepts the highest offer given that one of the offers is at least c_L. Likewise, the high-type seller accepts the highest offer of at least c_H.

Under this conjectured acceptance and disclosure strategy by the seller, we characterize buyer offer strategies for a given belief π in the offer stage game. In the second period, buyers update their beliefs over θ based on the seller's disclosure decision, the information disclosed, and the realization of the private signal s_i. For ease of exposition, we split the updating into two steps. Second-period buyers first update based on the disclosure decision and the information contained therein. We denote by $\pi_1^d := \mathbb{P}(\theta = H \mid d; \pi_0)$ the posterior belief following the disclosure decision d and prior belief π_0. After updating to π_1^d, second-period buyers then further update beliefs upon observing their private signals.

We find that buyer offer strategies are the same as those characterized in Lemma 1, with the exception that second-period buyers use the history of disclosure (and the disclosed information, if disclosure occurs) to update beliefs. To accommodate Lemma 1 to the voluntary regime, we consider any posterior belief prior to the second step above, i.e., before buyers update beliefs based on their private signals. In this way, our analysis in Lemma 1 applies to both the first and second period offer strategies. Specifically, we consider a generic "prior" belief of $\pi := \pi_1^d$ for the second period, and the prior belief $\pi := \pi_0$ in the context of the first period. We refer to the post-disclosure public belief in the second period (but before private signals have been observed) as that period's *initial* belief.

We see in Lemma 1 that buyers are more aggressive with their offers when beliefs are higher. An important question therefore is whether the seller is able to influence market beliefs with trading behavior and disclosure in an earlier period. Consider the offer game in period 1 for Cases 2–3, where only h-buyers target the high-type seller. If an offer of c_H or above arrives by at least one buyer, then the seller of either type accepts an offer with certainty in that period. Since trade took place, the seller can choose to disclose or withhold the trading price p_1. In this situation, disclosure of $p_1 \geq c_H$ implies that at least one buyer received signal h, resulting in a positive belief revision following disclosure for period-2 buyers. As such, the seller always discloses the trading price of c_H or above if trade took place.

In contrast, if both period-1 buyers observe signal ℓ, then the highest offer is v_L, for which only the L-seller accepts and the H-seller rejects. As noted previously, the H-seller cannot verifiably disclose that trade did *not* take place. Moreover, disclosure by the L-seller of $p_1 = v_L$ implies that both buyers in period 1 received the low signal ℓ with positive probability, resulting in a downward revision in beliefs by period-2 buyers following disclosure. Consequently, the L-seller conceals information regarding her trade in the first period and mimics the non-disclosure of the H-seller.

We note that since voluntary disclosure of the past transaction is a signaling activity, there are other, perhaps less plausible, equilibria, a property not uncommon in sequential games of incomplete information. For example, we can construct an equilibrium where the seller of either type withholds trade that occurred at a high price if we assign pessimistic off-path beliefs following disclosure of such trade.[11] We utilize a natural refinement based on deviation incentives, developed by Grossman and Perry (1986) and Farrell (1993), and employed in

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[11]Since disclosure is verifiable, disclosure of trade and the trading price pegs second-period buyer beliefs that trade took place at the disclosed price with probability one. However, buyers' inferences of the type θ are not restricted by the disclosure.

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the verifiable disclosure studies of Bertomeu and Cianciaruso (2018) and Glode et al. (2018). The Grossman-Perry-Farrell equilibrium (GPFE) refinement rules out equilibria where types can benefit from deviating from an on-path action, given that these types are conjectured to deviate. In other words, a given equilibrium fails the GPFE criterion if any deviator selects an off-path action, is "identified" correctly as a type that deviated, and receives a better payoff from this deviation than from the on-path action.

Proposition 1. *For any prior belief π_0, there exists a perfect Bayesian equilibrium of the voluntary disclosure regime of the following form.*

(i) *In the first-period offer stage, the θ-seller accepts the highest offer x_i that is at least c_θ. Offer behavior in the first period is as described in Lemma 1.*

(ii) *In Cases 1–3, at the start of $t = 2$ the seller of either type discloses the trade that occurred in the previous period if and only if the winning offer is c_H or higher.[12]*

(iii) *Disclosure always results in upward belief revision and non-disclosure results in downward belief revision by second-period buyers.*

(iv) *In the second period, offer behavior is as described in Lemma 1, where the "initial" belief is the belief following the seller's disclosure decision, π_1^d. The θ-seller accepts the highest offer x_i that is at least c_θ.*

Moreover, such an equilibrium is the essentially unique equilibrium characterization that survives the Grossman-Perry-Farrell criterion.

We have two countervailing effects following non-disclosure by the seller. Since non-disclosure encompasses rejection by the H-seller, the potential lack of trade by this type in the first period can positively improve beliefs of second-period buyers. Conversely, as noted above, non-disclosure implies that both period-1 buyers received the low signal with positive probability, which negatively impacts market beliefs. Interestingly, we find that, for any pre-signal belief π, the latter effect dominates and non-disclosure always results in a downward revision in beliefs of second-period buyers, thus indicating bad news (part (iii) of Proposition 1). This occurs because of mimicry by the L-seller of the H-seller's disclosure strategy; the market cannot determine if non-disclosure is due to lack of trade or due to concealing trade that actually occurred. This weakens the signaling value of rejection by the H-seller in the first period, resulting in the market placing greater weight on the event that

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[12]In Case 4, the seller is indifferent between disclosure and non-disclosure when the period-1 trade price is $\mathbb{E}[v_\theta | s_i = \ell, s_j = \ell; \pi_0] > c_H$. Since trade always occurs in this case, non-disclosure is always correctly interpreted as trade at the "low" price in Case 4. We assume without loss of generality and for ease of exposition that, when indifferent between disclosure and non-disclosure, the seller withholds trade.

two negative signals arrived in the first period. The net effect is thus a downward revision in buyer beliefs following non-disclosure in the voluntary regime.

Moreover, in Proposition 1, we additionally show that the equilibrium identified above is the essentially unique equilibrium that survives the GPFE criterion.[13] Additional details are provided in Appendix C.

While non-disclosure is always interpreted as bad news, an important, and perhaps efficiency-increasing, feature of the voluntary regime is that the L-seller always trades in the first period. In the following section, we characterize equilibrium in the mandatory regime and then analyze welfare comparisons between the two regimes.

3.3   Mandatory regime

In the mandatory disclosure regime, the seller must disclose whether trade occurred in the first period. Hence, unlike the voluntary regime, the L-seller can no longer conceal if trade took place at a price below c_H. The H-seller can thus use rejection in the first period to signal her type. Conversely, trade by the low-type seller at a price below c_H can indicate to second-period buyers that the seller is type L.[14] We therefore consider when the L-seller engages in mimicry of the high-type in the first period.

Analogous to the voluntary regime, we let second-period buyer beliefs following the first period, but before private signal realizations, be denoted as π_1 (i.e., the initial belief of that period). We denote the expected second-period payoff for a θ-seller who has rejected all first-period offers as $Q_\theta(\pi_1^R)$, where π_1^R denotes second-period buyers' beliefs upon rejection in the first period. If the L-seller accepts an offer of v_L in the first period, her total payoff after period 2 is $(1 + \delta)v_L$. If the L-seller does not accept an offer in the first period, her period-1 payoff is c_L and her continuation payoff is $\delta Q_L(\pi_1^R)$. For mimicry to occur in the first period, the L-seller must be at least as well off from rejection in the first period as she is from accepting v_L in that period and revealing herself to be of type L. The continuation value must therefore satisfy the following condition in equilibrium:

$$c_L + \delta Q_L(\pi_1^R) \geq (1 + \delta)v_L. \tag{3}$$

For the L-seller's acceptance strategy, we denote by $\phi \in [0, 1]$ the probability that the

[13]The equilibrium is essentially unique in the sense that any equilibrium that survives the GPFE criterion must take the form described in Proposition 1, but there is a continuum of (uncontroversial) off-path beliefs which support such an equilibrium.

[14]We later show that only the L-seller accepts offers below c_H in any equilibrium that survives the D1 criterion.

L-seller accepts an offer of v_L in the first period. As an initial property, we cannot have an equilibrium in which the L-seller accepts v_L with certainty in the first period, i.e., $\phi = 1$. In this case, upon rejection in the first period, second-period buyers must believe that the seller is a high type with probability one, resulting in offers of v_H. However, this implies that mimicry by rejecting offers is more valuable for the L-seller in the first period. Hence, we must have $\phi < 1$ in any equilibrium.

In Cases 1–3 discussed in Section 3.1, there is positive probability of the v_L-offer arriving in the first period, as the prior belief π_0 is sufficiently low in these cases. As in the voluntary disclosure regime, we have countervailing effects following two offers of v_L in the first period. However, as we will see shortly, in contrast to the voluntary regime, lack of trade in the first-period can lead to upward belief revision by second-period buyers. The first effect represents the *signaling value of transparency*. Because the L-seller accepts an offer of v_L with positive probability and the H-seller rejects such offers with probability one in the first period, this effect positively influences beliefs over θ for period-2 buyers. At the same time, lack of trade at a price of c_H or higher implies that both first-period buyers received a low signal with a higher likelihood. This effect has a negative impact on beliefs. To see which effect dominates, we consider the L-seller's ex ante trade incentive before offers arrive in time 1 based on the continuation value evaluated at the prior π_0.[15] We denote by π_1^D the belief after first-period acceptance of a high offer.[16]

- *Excessive rejection*: $(1+\delta)v_L < c_L + \delta Q_L(\pi_0)$. In this case, the L-seller ex ante prefers the continuation payoff Q_L (evaluated at π_0) over acceptance of v_L in the first period. The L-seller therefore has a strong incentive for mimicry through rejecting low offers in the first period. Due to the high mimicry, period-2 buyer posterior beliefs are revised *downward* following first-period rejection, $\pi_1^R < \pi_0$. That is, second-period buyers target the L-seller more in the second period than first-period buyers did in the first period (i.e., there is a higher weight on v_L in the mixed strategy distribution by period-2 buyers relative to period-1 buyers). Consequently, the negative information of two ℓ signals following the first-period rejection overtakes the positive information of rejection of low offers. As second-period buyers target the H-seller less frequently, this lowers the continuation value $Q_L(\pi_1)$ for the L-seller. We have two possibilities. The first is that condition (3)

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[15]In equilibrium, the L-seller's first-period trade incentive is determined by the second-period buyers' posterior belief π_1 following the acceptance or rejection decision in the first period. In the exposition, we discuss the L-seller's ex ante trading incentives in terms of the prior belief π_0 to present intuition for how the equilibrium posterior π_1 and the L-seller's first-period trading strategy are formed.

[16]We note that this posterior is the same as the posterior following acceptance and disclosure of trade at price $p_1 \geq c_H$ in the voluntary regime.

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holds with equality in equilibrium. In this case, beliefs are revised downward following rejection but the L-seller still accepts low offers with positive probability in equilibrium, $\phi \in (0, 1)$. This occurs when $(1 + \delta)v_L$ is not too low relative to $c_L + \delta Q_L(\pi_0)$. In the second case, condition (3) holds with strict inequality. Here, the L-seller rejects low offers with probability one and only accepts high offers. While $\phi = 0$ results in maximum downward revision following rejection, this can occur in equilibrium because period-2 buyers who have received the high signal may still target the H-seller with positive probability in the second period. This case occurs when $(1+\delta)v_L$ is low relative to $c_L + \delta Q_L(\pi_0)$.

- *Preference for acceptance*: $(1 + \delta)v_L \in [c_L + \delta Q_L(\pi_0), c_L + \delta Q_L(\pi_1^D))$. Here, the initial belief π_0 is such that ex ante the L-seller has a stronger incentive for accepting the low offer in the first period than for receiving the continuation payoff $Q_L(\pi_0)$ from rejection, evaluated at π_0. Consequently, the continuation payoff must be improved through second-period buyers targeting the H-seller with sufficiently high likelihood so that the L-seller begins rejecting offers in the first period. Because the acceptance probability of the L-seller is high, rejection of v_L offers in the first period results in overall positive belief revision by second-period buyers, i.e., $\pi_1^R > \pi_0$, and the signaling value dominates the negative information implied by the low offer in the first period. Consequently, second-period buyers target the H-seller frequently enough (i.e., making offers of c_H or higher) such that the continuation value following first-period rejection, $Q_L(\pi_1^R)$, rises to the point where condition (3) holds with equality. However, the signaling value is not strong enough (i.e., $\pi_1^R < \pi_1^D$) for the H-seller to reject a profitable high offer.

- *Stronger preference for acceptance*: $(1 + \delta)v_L \in [c_L + \delta Q_L(\pi_1^D), c_L + \delta v_H)$. In this final case, the L-seller has the strongest incentive to accept an offer of v_L in the first period. Period-2 buyers must therefore target the H-seller even more frequently in the second period to induce the L-seller to reject offers in the first period. This implies that beliefs following first-period rejection must be very high. Indeed, due to the L-seller's strong incentive to accept v_L, the signaling value is so dominant that rejection increases period-2 buyers' beliefs more than acceptance of a high offer (i.e., $\pi_1^R \geq \pi_1^D$). Consequently, the H-seller also begins to reject offers in the first period that she would otherwise accept, such as offers above c_H. Rejection of high offers further positively impacts second-period beliefs, as it implies a greater likelihood that a period-1 buyer received the high signal, resulting in higher second-period offers. To induce the H-seller to accept offers in the first period, buyers in that period must offer at least $c_H + \xi$ when targeting the H-seller,

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where $\xi \geq 0$ is determined such that the following condition holds:

$$c_H + \xi + \delta Q_H(\pi_1^D) = c_H + \delta Q_H(\pi_1^R). \tag{4}$$

The higher minimum offer targeting the high type makes it less worthwhile (or more costly) for the H-seller to reject high offers. As a consequence, no offer of $x \in [c_H, c_H + \xi)$ arrives in the first period, as this offer would only be accepted by the L-seller.

We note that, as in the voluntary regime, under certain conditions other equilibria can exist which are supported by an extreme off-path belief. For example, with high enough π_0, we can construct an equilibrium where both seller types accept v_L in the first period, supported by pessimistic off-path beliefs following rejection. As such, we utilize the D1 criterion (Cho and Kreps, 1987) to rule out implausible equilibria.[17]

We use the following notation to classify cases based on the initial (i.e., pre-signal) belief π: there exist cutoffs $\{\underline{\pi}, \overline{\pi}, \pi^\dagger\}$ such that for $\pi \leq \underline{\pi}$, the belief π corresponds to Case 1; $\pi \in (\underline{\pi}, \overline{\pi}]$ corresponds to Case 2; $\pi \in (\overline{\pi}, \pi^\dagger)$ corresponds to Case 3; and $\pi \geq \pi^\dagger$ corresponds to Case 4.[18] Finally, due to the additional richness of seller behavior and offer strategies in the mandatory regime, we derive offer strategies in Proposition B.1 of Appendix B. Buyer offer behavior closely resembles the strategies in Lemma 1; the difference is that, when the belief following first-period rejection is high, buyers must offer a minimum of $c_H + \xi$ when targeting the H-seller (instead of c_H), as noted above.

Proposition 2. *The first-period equilibrium in the mandatory regime is as follows. Buyers make offers according to Proposition B.1. If $(1 + \delta)v_L \in c_L + \delta[Q_L^-(\pi^\dagger), Q_L^+(\pi^\dagger)]$ (i.e., the rejection belief is at the boundary of Cases 3 and 4), where $Q_L^-(\pi^\dagger)$ and $Q_L^+(\pi^\dagger)$ are the left and right limits of Q_L at $\pi^\dagger$, then buyers mix between offering v_L and c_H so that condition (3) holds with equality. Furthermore, we have:*

(i) If $(1 + \delta)v_L < c_L + \delta Q_L(\pi_0)$, then the L-seller accepts the offer v_L with probability $\phi \in [0, 1)$, which is uniquely determined by equation (3) if interior, and she accepts any offer $x_i \geq c_H$ with probability one (or the highest of such offers). The H-seller accepts

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[17]The implausible equilibrium noted above fails even without D1 when π_0 is in Case 1 or Case 2, as the H-seller is met with a low posterior belief (equal to π_1^{ND}) after acceptance of v_L. Since this seller's payoff is c_H in the second period regardless of what she does in the first period, she would instead reject low offers in the first period. We provide more detail regarding equilibrium refinement in Appendix C.

[18]Specifically, the cutoffs are defined as follows:

$$\underline{\pi} := \frac{\overline{\pi}(1 - q)}{q - \overline{\pi}(2q - 1)}, \quad \overline{\pi} := \frac{c_H - v_L}{v_H - v_L}, \quad \pi^\dagger := \frac{q^2 \overline{\pi}}{(1 - q)^2 + (2q - 1)\overline{\pi}}.$$

the highest offer x_i with probability one if and only if $x_i \geq c_H$. The belief after rejection of the offer v_L is lower than the prior: $\pi_1^R < \pi_0$.

(ii) If $(1+\delta)v_L \in [c_L + \delta Q_L(\pi_0), c_L + \delta Q_L(\pi_1^D))$, then the seller's acceptance behavior is the same as in (i) above. The belief after rejection of offer v_L is (weakly) higher than the prior: $\pi_1^R \geq \pi_0$.

(iii) If $(1+\delta)v_L \in [c_L + \delta Q_L(\pi_1^D), c_L + \delta v_H)$, then the minimum offer targeting the H-seller is $c_H + \xi$, where ξ is determined by equation (4). The L-seller's acceptance strategy is the same as in Cases (i) and (ii) above. The H-seller accepts the highest offer x_i if and only if $x_i \geq c_H + \xi$. The belief after rejection is higher than the prior: $\pi_1^R \geq \pi_1^D > \pi_0$.

In the second period, the L-seller accepts the highest offer x_i if and only if $x_i \geq c_L$ and the H-seller accepts the highest offer x_i if and only if $x_i \geq c_H$. Buyer offer strategies are determined as in Proposition B.1, with initial belief π_1. Moreover, such an equilibrium is the essentially unique equilibrium characterization that survives the D1 criterion.

3.4   Welfare and efficiency: Mandatory vs. voluntary disclosure

We now compare the welfare and trading efficiency implications of the two regimes. To measure welfare, we consider the total expected payoffs of the seller and both sets of buyers in each regime. We begin with the statement of the welfare result:

Theorem 1. *Welfare between the two regimes has the following properties:*

(i) (Voluntary Dominates Mandatory) *Suppose that $\pi_0 \in [\bar{\pi}, \pi^\dagger)$. That is, the initial belief is high-intermediate (Case 3). Then, welfare is strictly higher under the voluntary regime when:*

   (i.a) *c_L and π_0 are high enough;[19]*

   (i.b) *c_L is low enough and π_0 is high enough.[20]*

(ii) (Mandatory Dominates Voluntary) *Suppose that $\pi_0 \leq \bar{\pi}$. That is, the initial belief is low-intermediate or low (Case 2 or 1). Then, welfare is strictly higher under the mandatory regime, if*

   (ii.a) *$\pi_0 \in (\underline{\pi}, \bar{\pi})$ (Case 2) and $(1+\delta)v_L \leq c_L + \delta Q_L(\pi_1^D)$;*

   (ii.b) *$\pi_0 \leq \underline{\pi}$ (Case 1).*

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[19]To be precise, there exist $c_L^* < v_L$ and $\pi^* < \pi^\dagger$ such that, for all $c_L \in (c_L^*, v_L)$ and $\pi_0 \in (\pi^*, \pi^\dagger]$, welfare is strictly higher under the voluntary regime.

[20]See Appendix B.5 for precise conditions.

(iii) (Indifference) *Suppose that $\pi_0 \geq \pi^\dagger$. That is, the initial belief is high (Case 4). Then, welfare is equal in the two regimes.*

We see in part (i) of Theorem 1 that voluntary disclosure is welfare-improving relative to mandatory disclosure under certain conditions. This is perhaps surprising given that mandatory disclosure provides more trade information to buyers, while voluntary disclosure entails relevant information being strategically suppressed by the seller.

Under mandatory disclosure, we observe two important cases where efficiency loss becomes severe. In the first case, the L-seller rejects low offers in the first period with a high probability, including with probability one. In contrast, under the voluntary regime, trade is always efficient in the first period—rejection only occurs when both first-period buyers target the L-seller and the true state is $\theta = H$. Voluntary disclosure therefore attenuates the low-type seller's incentive for excessive first-period rejection and is thus superior to mandatory in this situation, corresponding to condition (i.a) of Theorem 1.[21,22]

The second case of severe inefficiency under mandatory disclosure occurs when the H-seller rejects high offers below $c_H + \xi > c_H$ in the first period. Welfare destruction occurs here because first-period buyers target the H-seller with lower probability. In the worst case, when ξ is too high, h-buyers target the H-seller with probability zero and only make low offers.[23] This situation arises when the L-seller has a strong ex ante preference for first-period acceptance of offer v_L, corresponding to condition (i.b) of Theorem 1. Conversely, this situation cannot arise under the voluntary regime; while the L-seller always accepts offers of v_L, the seller never discloses trade that occurs at price v_L. As such, the H-seller always accepts offers of at least c_H, resulting in welfare-improvement relative to the mandatory

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[21]The conditions that c_L and the initial belief π_0 need to be sufficiently high indicate the strong preference for rejection in the mandatory regime—a high c_L implies that rejection of v_L in the first period is not as costly for the L-seller, while a high π_0 indicates that the seller is likely to receive a high offer in the second period, even after first-period rejection. We additionally note that, in the situation where the L-seller rejects low offers with probability one in the mandatory regime, the rejection belief is the same as the non-disclosure belief (i.e., $\pi_1^R = \pi_1^{ND}$) implying that the H-seller does not have any difference in second-period trade probability under either regime in this case.

[22]We note that the welfare difference between voluntary and mandatory disclosure is not monotone in the first-period trade probability with the L-seller. This is because, while the probability of trade decreases as c_L increases in the mandatory regime, the welfare advantage of voluntary over mandatory disclosure also decreases in c_L (trade with the L-seller entails surplus of $v_L - c_L$). Our proof approach in Theorem 1 focuses on establishing that there exists a non-degenerate range of high c_L values for which the L-seller rejects low offers with probability one in the mandatory regime.

[23]For example, suppose π_0 is in Case 3. If the equilibrium ξ must be very high to induce acceptance by the H-seller in the first period, we can have that $c_H + \xi > \mathbb{E}[v_\theta | s_i = h; \pi]$. In this situation, the H-seller rejects all offers that are below $c_H + \xi$, but posterior beliefs after observing signal h are not high enough for an h-buyer to offer $c_H + \xi$. Consequently, h-buyers only target the L-seller, and thus we are effectively in Case 1, where all buyers only target the L-seller.

regime.

In part (ii) of Theorem 1, when initial beliefs are sufficiently low, the mandatory regime is welfare-enhancing relative to the voluntary regime. Unlike in part (i), here the transparency allows the H-seller to build her reputation through early rejection, as acceptance occurs sufficiently often by the L-seller in period 1. Consequently, total welfare is higher under the mandatory regime as the H-seller trades more often in the second period. Finally, in part (iii) of Theorem 1, initial beliefs are so optimistic that both types of sellers accept offers in the first period with probability one. Moreover, the probability of trade is the same in the second period under both regimes. This implies that welfare is the same in both regimes.[24]

The above discussion implies that the voluntary regime exhibits greater trading frequency or volume when the mimicry incentive is high. We find that this is indeed the case; under voluntary disclosure, trade of the low asset always occurs in the first period, while this occurs with probability less than one in the mandatory regime. While the lack of signaling can result in lower trading volume of the high asset in the voluntary regime, this is offset by the high trading volume of the low asset, when the mimicry incentive of the L-seller is strong.

Corollary 1. *The voluntary regime exhibits weakly higher trading volume than the mandatory regime when π_0 and c_L are sufficiently high.*

3.5 Discussion of results and implications for transparency regulation

The value of voluntary disclosure relative to mandatory dissemination stems from the property that the seller's ability to strategically suppress negative trade information removes the destructive signaling incentives of rejecting profitable trade that arise under mandatory disclosure. At the same time, the seller can still reveal relevant information to new buyers. Indeed, destructive signaling (i.e., rejection of offers by the low type) only occurs in the mandatory regime. As such, when the mimicry incentive is strong, voluntary disclosure permits less costly information transmission than under mandatory disclosure. Hence, despite mandatory disclosure often being more informative to second-period buyers, this benefit can be outweighed by destructive signaling incentives inhibiting profitable trade.

The second central element of our setting is (imperfect) private information by buyers, which plays two distinct roles. First, disclosure of trade becomes more informative with

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[24]See Appendix Section A.1 for additional discussion on this case; in Section A.1 we also examine the welfare comparison with the no-disclosure benchmark introduced in Section 3.1.

buyer private signals, as this information is conveyed indirectly through the trade price.[25] For example, this property implies that disclosure of sales at a high price is indicative of high asset quality. Voluntary disclosure of trade at a high price is thus informative for second-period buyers—allowing for informative communication through disclosure despite the lack of signaling through offer rejection.[26] Second, buyer private information amplifies signaling incentives by the low-type seller in the mandatory regime; indeed, it is precisely because of buyer private information that can drive the low-type seller to reject low offers with probability one in the first period, leading to a large welfare loss.

In sum, the interplay between voluntary disclosure, which allows for less costly communication, and buyer private information, which makes disclosure informative, are the economic forces that drive the welfare improvement of voluntary disclosure relative to a mandatory regime (as well as relative to an opaque regime; see Appendix Section A.1).

The results of the model have implications for transparency regulation of trades in financial markets, particularly over-the-counter (OTC) markets. While the conventional wisdom is that information is better for trade, our analysis implies that the equilibrium effects of mandating disclosure can alter seller behavior in ways that are destructive to trade. Furthermore, mandatory disclosure can be improved upon by a voluntary regime when signaling incentives are strong. In what follows, we discuss applications of our model and the related empirical literature.

In the U.S. corporate bond market, the SEC approved the mandatory public dissemination of OTC transactions—including trade prices and volumes—through the Trade Reporting and Compliance Engine (TRACE) platform, beginning in 2002. Asquith et al. (2019) find that the overall number of trades declined by 11.7% following the introduction of TRACE, with most of this decrease being driven by more illiquid, higher-yield bonds, which exhibited a 71.1% decrease in the number of trades.[27] When comparing a matched sample of U.S. bonds after TRACE and German bonds prior to any transparency regulation, Gündüz et al. (2023) similarly find lower liquidity in the U.S. market. In European OTC markets, the EU regulation Markets in Financial Instruments Directive (MiFID II) expanded transparency requirements of bond trades, beginning in 2018. Meli et al. (2024) document higher

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[25]More precisely, second-period buyers update their beliefs of asset value based on the posterior distribution of signals observed by first-period buyers.

[26]Related to this point, in Appendix Section A.3, we examine our setting where the manager can only disclose that trade occurred and not the trade price. This analysis highlights the importance of disclosure of both trade and the price at which trade took place.

[27]Bessembinder et al. (2006) examine only the initial set of bonds that were exposed to TRACE and find positive effects; the negative effects documented in Asquith et al. (2019) are driven by bonds exposed to TRACE in later phases.

bid-offer spreads for larger trades following MiFID II.

While TRACE implementation entailed mandatory dissemination of trade prices and volumes, our results imply that, for assets where signaling incentives are severe, voluntary disclosure can be advantageous in facilitating trade. More specifically, as established in Section 3.3, when c_L—the L-seller's payoff following no trade in a period—is high, the low-type seller is less inclined to sell at the low price in the mandatory regime and prefers to mimic through rejection. In the context of the corporate bond market, we can interpret c_L as capturing the inverse of dealer inventory costs. For example, an asset with a lower inventory cost corresponds to a higher reservation payoff (i.e., a lower cost from forgoing trade), and thus is represented by a higher c_L.[28] Our results in Theorem 1 imply that, when adverse selection frictions are not too severe, voluntary disclosure of trade can be advantageous for both low- and high-inventory cost bonds.

Similar to the corporate bond market, the decentralized U.S. municipal bond market likewise experienced a gradual shift towards trade transparency, beginning in 1995 and becoming fully transparent in 2005. Studying this setting, Brancaccio et al. (2020) find that trading of uninsured municipal bonds falls relative to insured bonds, while Schultz (2012) documents a decrease in dispersion but also a decrease in liquidity (due to higher markups) for larger trades. Uninsured bonds are likely to have higher inventory costs for dealers. Continuing with our interpretation of c_L above, Theorem 1 (i.b) implies that welfare can be improved under voluntary disclosure of trade for these types of bonds, given that adverse selection frictions are not too severe. This latter condition can correspond to, for example, a low historical default risk by the municipality. Conversely, Theorem 1 (ii.b) implies that, when adverse selection is the most severe (i.e., Case 1), such as for bonds issued by municipalities with historically high default rates, mandatory disclosure is welfare-enhancing relative to both voluntary disclosure and the opaque regime (analysis of the latter is provided in Section A.1).

4   Long horizon

Our baseline setting considers a parsimonious two-period model that captures the main economic insights of voluntary disclosure of trade information relative to a mandatory regime.

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[28]Inventory costs can include, for example, the cost of hedging price risk of the bond, funding and carry costs, regulatory capital costs, and opportunity costs. Furthermore, lower-quality bonds typically have higher inventory costs, which is consistent with inventory costs being lower for H-sellers with higher quality bonds, in line with the assumption that $c_H > c_L$. A similar interpretation is used in Duffie et al. (2005).

We now extend our baseline setting to more than two periods.

We consider both the case of $T \geq 3$ periods and the infinite horizon limit, $T \to \infty$. As in the baseline setting, the seller's per-period payoff upon acceptance of an offer in period t is $p_t \delta^{t-1}$, while upon rejection the seller's payoff is $c_\theta \delta^{t-1}$, where $\delta \in (0,1)$. We continue to impose Assumptions 1 and 2. We assume that current buyers in period t are able to observe the public history of disclosure or lack thereof (under the voluntary regime) or trade (under the mandatory regime) by the seller in all periods prior to t. In particular, at the beginning of period t, buyers observe the public history of disclosure up to period $t-1$ and the period-t disclosure outcome of the prior period's possible trade activity. We denote the public information set of period-t buyers after the disclosure stage, but before each buyer's private signal arrives and offers are made, as $h^t = (m_1, \ldots, m_{t-1})$, where m_{t-1} denotes the disclosure outcome at the beginning of period t in each regime. The public belief in period t based on the public disclosure history h^t is denoted as π_{t-1}. Recall that we refer to the public belief following disclosure, but before private signals are observed, as the *initial* belief for that period.

Below, we analyze equilibrium and welfare comparisons between the two regimes. We present additional analysis related to long-run belief convergence in Appendix Section A.2, which provides further insights into the welfare properties of the two regimes over a long horizon.

4.1 Equilibrium of the Long-Horizon Model

We first show that the equilibrium structure of the two-period model naturally extends to the long-horizon setting under both regimes. Intuitively, in any period $t < T$, given fixed equilibrium strategies from period $t+1$ onward, the game reduces to the two-period model. We look for a stationary Markov perfect equilibrium in which buyers' offer strategies and the seller's acceptance and disclosure strategies in each period depend only on the initial public belief π_{t-1} (Duffie et al., 1994; Maskin and Tirole, 2001; He and Sun, 2017). The formal analysis involves additional technical details; we relegate these to the Appendix and discuss the key results below.

Voluntary Regime

We first examine the voluntary regime. In the final period T, the equilibrium is the same as in the static case (i.e., a single period of the no-disclosure benchmark in Section 3.1). Consider period $T-1$. Given the initial belief in period $T-1$, denoted as π_{T-2}, the game

is the same as the two-period model. In particular, the H-seller only accepts offers that are weakly higher than c_H, and the L-seller accepts any offer at least as high as c_L but conceals trades at prices below c_H. In period $T-2$, given the initial belief π_{T-3} and the above equilibrium strategies in period $T-1$, the game again reduces to the two-period model, where the continuation value from period $T-2$ onward is used as the "second-period" payoff in the two-period model.

By backward induction, the equilibrium strategy takes the same form in each period as in the two-period model. Therefore, it suffices to keep track of disclosure outcomes $d_{t-1} \in \{D, ND\}$, with $d_{t-1} = D$ indicating disclosure and $d_{t-1} = ND$ indicating non-disclosure of the period-$(t-1)$ trade. As such, we let $m_{t-1} = d_{t-1}$ under the voluntary regime. In period t, buyers update their pre-signal beliefs to $\pi_{t-1} = \mathbb{P}(\theta = H \mid h^{t-1}, d_{t-1})$ upon observing the disclosure outcome.

Proposition 3. *Consider the voluntary regime with $T \geq 3$ periods. A stationary Markov perfect equilibrium exists with the following strategy profile. In each period $t \leq T$, buyers make offers according to Lemma 1 with π_{t-1} used as the initial belief. The H-seller accepts the highest offer that is at least c_H in each period and discloses whenever trade occurs. The L-seller accepts the highest offer that is at least c_L in each period and discloses trades that occurred at prices of at least c_H.[29] This strategy profile also constitutes a stationary Markov perfect equilibrium in the limit as $T \to \infty$.*

The equilibrium possesses a Markov structure. In each period, the initial belief determines the offer strategies of buyers as well as the seller's acceptance and disclosure strategies. As a result, the equilibrium extends to $T \to \infty$: the seller accepts any on-path offers, and the L-seller withholds trade at v_L; disclosure results in an upward revision of beliefs, while non-disclosure leads to a downward revision.

Mandatory Regime

We next examine the mandatory regime. As in the voluntary case, backward induction shows that the equilibrium structure in each period $t < T$ is the same as in the two-period model. As such, we let $m_{t-1} = a_{t-1}$ be the relevant disclosure outcome under the mandatory regime. We simplify notation by capturing the relevant information in the seller's acceptance decision, such as whether the seller accepted a "high" or "low" offer, or whether the seller

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[29]As in the two-period setting, in Case 4 we assume without loss of generality that the seller withholds disclosure when indifferent (trade always transpires in Case 4, so non-disclosure is interpreted as trade at the "low" price).

rejected all offers. Specifically, we let $a_{t-1} \in \{R, p_{\text{high}}, p_{\text{low}}\}$ denote the seller's acceptance or rejection decision in period $t - 1$. Here, R indicates rejection, p_{high} indicates that the seller accepted a "high" offer that targets the H-seller, and p_{low} indicates acceptance of a "low" offer that targets the L-seller.[30] Let $Q_\theta^{(T-t)}(\pi)$ be the θ-seller's continuation payoff when $T - t$ periods are remaining with initial belief π. When $T - t$ periods are remaining, the L-seller's equilibrium strategy in that period (i.e., the probability of accepting the v_L-offer) is determined by the following condition, analogous to condition (3) in the two-period model:

$$c_L + \delta Q_L^{(T-t)}(\pi_t^R) \geq \sum_{\tau=0}^{T-t} \delta^\tau v_L, \tag{5}$$

where $\pi_t^R = \mathbb{P}(\theta = H \mid h^t, a_t = R)$ is the belief after rejection. When condition (5) holds with equality, the L-seller mixes between accepting and rejecting a low offer; otherwise, she rejects the low offer with probability one. Buyers' offer strategies (i.e., the adjustment parameter) are similarly characterized by the following condition, analogous to condition (4) in the two-period model:

$$c_H + \xi^{(T-t)} + \delta Q_H^{(T-t)}(\pi_t^D) = c_H + \delta Q_H^{(T-t)}(\pi_t^R), \tag{6}$$

where $\pi_t^D = \mathbb{P}(\theta = H \mid h^t, a_t = p_{\text{high}})$ is the belief after acceptance of a high offer (the belief after acceptance of offer v_L is zero).[31] Condition (6) is relevant only when $\pi_t^R > \pi_t^D$ and corresponds to the "stronger preference for acceptance" case of Section 3.3 (otherwise $\xi^{(T-t)}(\pi_{t-1}) = 0$).

Proposition 4. *Consider the mandatory regime with $T \geq 3$ periods. A stationary Markov perfect equilibrium exists with the following strategy profile. In period T, the equilibrium is the same as in the static case with initial belief π_{T-1}. In each period $t < T$, the equilibrium takes the following form: buyers make offers according to Proposition B.1 with π_{t-1} used as the initial belief; the H-seller accepts the highest offer that is at least $c_H + \xi^{(T-t)}$; and the L-seller accepts v_L with probability $\phi^{(T-t)}(\pi_{t-1}) \in [0, 1)$ and accepts all offers $x_i \geq c_H$ with probability one (or the highest of such offers). The values of $\phi^{(T-t)}(\pi_{t-1})$ and $\xi^{(T-t)}(\pi_{t-1})$ are determined by conditions (5) and (6).*

Furthermore, in the limit as $T \to \infty$, a stationary Markov perfect equilibrium of the same

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[30]Specifically, in Cases 1–3, $p_{\text{low}} = v_L$ and $p_{\text{high}} \geq c_H$. In Case 4, $p_{\text{low}} = \mathbb{E}[v_\theta \mid s_i = \ell, s_j = \ell; \pi] > c_H$ and $p_{\text{high}} \geq \mathbb{E}[v_\theta \mid s_i = \ell, s_j = \ell; \pi]$.

[31]Since the belief updating rule is the same under the voluntary regime, we use the notation π_t^D to emphasize this fact.

form exists.[32]

4.2 Welfare Comparison in the Long Horizon

The following theorem establishes that voluntary disclosure can be welfare-improving relative to mandatory disclosure under conditions analogous to those in Theorem 1 (i.a).[33] We evaluate welfare as the planner's present discounted value of total surplus, which is a standard approach for welfare analysis in dynamic settings with both long- and short-lived agents (e.g., Liu and Skrzypacz, 2014; Board and Skrzypacz, 2016; Vellodi, 2018; Kaya and Roy, 2022, 2024; Deng et al., 2024; Kovbasyuk and Spagnolo, 2024; Pei, 2024).[34]

Theorem 2. *Let $T \geq 3$ and $\pi_0 \in [\overline{\pi}, \pi^\dagger)$. Suppose that π_0 and c_L are high enough.[35] Then, there exists a cutoff level of the discount factor, denoted by $\overline{\delta} \in (0, 1)$, such that the voluntary regime gives strictly higher welfare than the mandatory regime for any $\delta \leq \overline{\delta}$. This result also holds for $T \to \infty$.*

We observe that the economic forces driving our two-period results are preserved in the long-horizon; total welfare can be improved in voluntary disclosure as this can mitigate the L-seller's welfare-destroying excessive rejection. The additional condition in Theorem 2 is that the discount factor must be sufficiently low. This condition arises because mandatory disclosure becomes efficient as the horizon lengthens. Specifically, when $T \to \infty$, beliefs converge to the true state, indicating that trade becomes efficient in the mandatory regime. In contrast, this efficiency convergence only occurs in the low state in the voluntary regime. (See Appendix Section A.2 for details.)

However, the earlier periods in the mandatory regime still carry the welfare losses from excessive rejection by the L-seller, due to the same reasoning as in Section 3.3. At the same time, trade continues to occur in early periods under the voluntary regime. Consequently,

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[32]That is, given the initial belief π in each period, buyers bid according to Proposition B.1; the H-seller accepts only offers that are weakly higher than $c_H + \xi(\pi)$; and the L-seller accepts v_L with probability $\phi(\pi) \in [0, 1)$. The values of $\xi(\pi)$ and $\phi(\pi)$ are determined by the following conditions: $c_L + \delta Q_L(\pi^R) \geq v_L/(1 - \delta)$ and $c_H + \xi + \delta Q_H(\pi^D) = c_H + \delta Q_H(\pi^R)$, where π^D and π^R are disclosure beliefs and rejection beliefs, respectively. The continuation payoffs are the limit of the finite-horizon case: $Q_\theta \coloneqq \lim_{T \to \infty} Q_\theta^{(T)}$.

[33]We note that the inefficiency under the mandatory regime corresponding to the "stronger preference for acceptance" case in Section 3.3 vanishes as the time horizon lengthens. When the probability of acceptance of low offers by the L-seller is very high, beliefs improve significantly following a first-period rejection and trade becomes more efficient. This effect is intensified in the long horizon as the gains from early belief improvement carry over to subsequent periods.

[34]For simplicity, we assume that the planner and long-lived seller share a common discount factor. This discounting approach to examine ex ante welfare is equivalent to considering the undiscounted total surplus but assuming that the game ends with probability $1 - \delta$ in each period, as in Pei (2024).

[35]The cutoffs are the same as those of Theorem 1 (i.a).

if the discount factor is sufficiently low, the welfare gains from early efficient trade under voluntary disclosure can outweigh the efficiency gains from belief convergence in the long horizon achieved under the mandatory regime.

This result is perhaps surprising given that a lower discount factor naturally induces the L-seller to accept early offers of v_L more frequently in the mandatory regime, thereby improving efficiency. In particular, as the L-seller discounts the future more heavily, she places less weight on buyer beliefs in the next period, thus resulting in greater acceptance of v_L in early offers. This additional force results in greater efficiency of trade under the mandatory regime and should minimize the welfare difference between mandatory and voluntary disclosure. However, we find that inefficiency persists under mandatory disclosure, even when δ is low, and does not decrease at the same rate as δ decreases. This analysis emphasizes that saving the signaling costs under voluntary disclosure are a first-order driver of the welfare benefits.

5   Concluding remarks

In this study, we examine a repeated trade setting where buyers receive private signals and the seller can voluntarily disclose past trade information. Despite the fact that strategic disclosure results in a more opaque information environment for buyers, voluntary disclosure can be welfare-enhancing relative to a mandatory disclosure regime where buyers always observe past trade behavior. This occurs because, when the low-type seller's mimicry incentive is strong, an unfavorable equilibrium emerges in the mandatory regime whereby the low-type seller rejects early low offers, but this early rejection is not met favorably by second-period buyers and instead results in a downward revision of beliefs over the seller's quality. The voluntary disclosure regime improves upon this situation, as it removes the low-type seller's ability to mimic a higher type by rejecting offers. As such, the low-type seller always trades at the low offer in every period, even when this is the highest offer.

Our setting can be extended in a number of dimensions. One possibility is to allow impersistence in the seller's type, which we explore in Appendix A.4. Allowing for impersistence in asset quality weakens the mimicry incentive of the low-type seller, thus reducing the edge of voluntary disclosure. Indeed, as the the asset becomes perfectly impersistent, disclosure in either regime becomes irrelevant. However, with sufficient persistence, the results of the model are not substantively affected. Interestingly, numerical exercises show (available upon request) that the degree of impersistence possible is increasing in the low-type seller's reser-

vation payoff. This is due to the strengthened mimicry incentive by the low-type seller when reservation value is high, resulting in the welfare gap holding for a greater degree of persistence. Another avenue is to allow for stochastic buyer arrival, explored in Appendix A.5. Second-period buyers are unable to determine if trade did not occur because of rejection or because buyers did not arrive in the first period. We similarly find that equilibrium features are preserved when search frictions are not severe.

Our model of repeated trade has applications to financial markets, particularly over-the-counter markets. Various OTC markets have been subject to increased post-trade transparency regulations in recent years, including a number of debt securities markets through TRACE, the European bond market through MiFID II, and the municipal bond market through MSRB Rule G-14. The empirical literature examining the impacts of these regulations has been mixed, with studies generally documenting both positive and negative effects of increased transparency depending on factors such as liquidity or yield of the asset. Our setting sheds light on possible improvements to transparency regulations, where permitting voluntary disclosure of past trade for certain asset types can improve trade frequency and welfare. In particular, when interpreting parameters of our model in the context of the OTC bond market, our results imply that voluntary disclosure can be advantageous relative to mandatory disclosure when adverse selection frictions are not too severe for high- and low-inventory cost bonds.

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Appendix

A   Additional analyses

In this section, we present additional analyses and extensions of the model.

A.1   Welfare comparison to the no-disclosure benchmark

Recall that in Section 3.1, we presented the equilibrium of the special no-disclosure benchmark case where the seller is unable to communicate trade information. We refer to the no-disclosure benchmark as the "opaque regime" for expositional ease. Based on the previous analysis in Section 3.4, we might expect the opaque regime to be attractive for reasons similar to the voluntary regime, whereby the lack of disclosure curbs the L-seller's ability to engage in destructive mimicry. However, we find that voluntary disclosure dominates the opaque regime as well, under similar conditions in which it dominates the mandatory regime, established in part (i) of Theorem 1.

In the first period, both the opaque and voluntary regimes have the same welfare—trade always transpires for the L-seller. In the second period of the voluntary regime, the seller either discloses the trading price (if trade occurred) or makes no disclosure (either strategically or if no trade occurred). In either event, the disclosure outcome is informative for second-period buyers. Disclosure of trade at price $p_1 \geq c_H$ in the voluntary regime results in upward updating of beliefs and a greater likelihood of trade in the second period. This leads to a welfare improvement of the voluntary regime over the opaque regime. Conversely, a lack of disclosure can harm welfare if the seller is the high type, as the trade probability declines. We find, however, that this negative effect from non-disclosure is outweighed by the positive effect from disclosure when initial beliefs π_0 are sufficiently high; even following non-disclosure, there is still a sufficient likelihood that h-buyers target the H-seller in the second period when π_0 is high enough. This implies that strategic disclosure is welfare-improving relative to the situation where the seller is unable to communicate past trade information.

Additionally, the mandatory regime dominates the opaque regime when the initial belief π_0 is sufficiently low. With low beliefs, the adverse selection problem is severe and buyers do not target the high-type seller. Mandatory disclosure is advantageous in this regard as the credible lack of trade in the first period allows the seller to improve second-period buyer beliefs, thus increasing the incidence of trade in that period.

Finally, we find that the opaque regime welfare (weakly) dominates both the voluntary and mandatory regimes when beliefs are very optimistic (i.e., Case 4 where $\pi_0 \geq \pi^\dagger$). First note that, in this case, trade always occurs for both seller types in the first period in both the voluntary and mandatory regimes. However, trade at the "low" price can still occur in the first period—where both buyers receive the ℓ-signal—and this situation lowers beliefs of second-period buyers in both the voluntary and mandatory regimes. For π_0 close to $\pi^\dagger$, this belief revision can be significant enough that the pre-signal belief of second-period buyers is lowered to Case 3. Since the posterior $\pi_1 < \pi^\dagger$, there is positive probability that second-period buyers do not target the H-seller and that trade does not occur, thus reducing welfare. Welfare in both the voluntary and mandatory regimes are equal in this situation, as

both regimes experience the same downward revision following trade at the low price in the first period (where buyers infer from non-disclosure in the voluntary regime that the trading price was low, as trade always occurs in Case 4). In contrast, this situation cannot arise in the opaque regime under optimistic beliefs; because second-period buyers do not observe any history of disclosure, beliefs continue to be high in the second period and trade always occurs for both types of the seller.

Proposition A.1. *We have the following welfare properties with respect to the opaque regime under which the seller is unable to communicate:*

(i) *Welfare is strictly higher under the voluntary regime than in the opaque regime when $\pi_0 \in (\pi^*, \pi^\dagger)$, where π^* is from Theorem 1.*

(ii) *Welfare is strictly higher under the mandatory regime than under the opaque regime for $\pi_0 \leq \underline{\pi}$.*

(iii) *Welfare is weakly higher under the opaque regime than the voluntary or mandatory regimes when $\pi_0 \geq \pi^\dagger$.*

Proof. See the proof of Theorem B.2, which combines the statements of Theorem 1 and Proposition A.1. □

A.2 Long-Run Behavior of Beliefs

In this appendix, we explore the belief convergence of the two regimes as $T \to \infty$. As we focus on long-run belief convergence, we define the following scenarios. In an *UP* cascade, remaining buyers make offers that target the high-type seller with probability one, regardless of their private information. Likewise, in a *DOWN* cascade, buyers only target the low-type seller (i.e., the only offer is v_L), again regardless of their private information. Once buyers perfectly learn the seller's type, a cascade ensues. However, cascades can also occur without perfect learning. We say that an *UP cascade without learning* occurs when an *UP* cascade occurs but the belief does not reach one. The definition of a *DOWN cascade without learning* is analogous. We present results separately for each regime. The results in this section are used in the welfare analysis of the long horizon.

In the voluntary regime, buyers do not observe disclosure of trade at a price below c_H, as disclosure of such trade implies that the seller is a low type with probability one, inducing a *DOWN* cascade. This would suggest that *DOWN* cascades are unlikely to occur in the voluntary regime. However, once beliefs become sufficiently low, buyers target the low-type seller regardless of their private information. Specifically, once Case 1 is reached (as described in Lemma 1), buyers only offer v_L. As such, Case 1 is an absorbing state—beliefs cannot improve if both buyers only offer v_L in every period.

Conditional on $\theta = L$, a *DOWN* cascade without learning occurs with positive probability for any finite T and occurs with probability one when $T \to \infty$. Since buyer private signals are correlated with the underlying state θ, in the long horizon buyers will more often receive the low signal ℓ. As a result, a seller of type L is less often able to disclose trade at a price of c_H or higher when T is large. Moreover, in the limit as the time horizon becomes infinitely long, non-disclosure occurs sufficiently often so that beliefs eventually drop to Case 1, resulting in a *DOWN* cascade without learning occurring with certainty. The absorbing nature of

Case 1 also implies that buyers never perfectly learn the seller's type, even when $T \to \infty$. When the initial belief begins in Case 1, no information is revealed on the equilibrium path.

Interestingly, conditional on $\theta = H$, a *DOWN* cascade without learning can occur with positive probability for both $T < \infty$ and $T \to \infty$. Because the *H*-seller cannot use rejection as a signaling mechanism in the voluntary regime, a series of low signals by buyers, resulting in consecutive non-disclosures, can push beliefs downwards to the point where the posterior belief reaches Case 1. As Case 1 is an absorbing state, even if remaining buyers receive signal h, the public belief does not recover since no buyers target the *H*-seller. However, as $T \to \infty$, an *UP* cascade with learning can also occur with positive probability, conditional on $\theta = H$. Learning remains possible because buyers can receive the high signal sufficiently often that Case 1 is never reached.

We define $T_k^{\mathcal{D}}$ to be the minimum number of periods for a *DOWN* cascade to occur (i.e., for the belief to reach Case 1) starting from π_0 in Case k. We summarize the above findings in the following proposition.[36]

Proposition A.2. *Consider the voluntary regime. Regardless of the initial belief $\pi_0 \in (0,1)$, a DOWN cascade occurs only without learning. Moreover, the equilibrium exhibits the following properties:*

(i) *Suppose π_0 is Case 1. A DOWN cascade without learning occurs with probability one.*

(ii) *Suppose π_0 is Case k with $k \in \{2,3,4\}$.*

- *Suppose $\theta = H$. If $T \geq T_k^{\mathcal{D}}$, a DOWN cascade without learning occurs with positive probability. In addition, as $T \to \infty$, then an UP cascade with learning occurs with positive probability.*

- *Suppose $\theta = L$. If $T \geq T_k^{\mathcal{D}}$, a DOWN cascade without learning occurs with positive probability. In addition, as $T \to \infty$, then a DOWN cascade without learning occurs with probability one. An UP cascade never occurs.*

Under the mandatory regime, conditional on $\theta = L$, a *DOWN* cascade also occurs. However, unlike the voluntary regime, a *DOWN* cascade is always due to learning. Upon acceptance of an offer v_L in the mandatory regime, future buyers correctly believe that the seller is of type L with probability one. Because the *L*-seller accepts v_L with positive probability (when the continuation value becomes low enough) when mimicking the high type, acceptance of the low offer eventually occurs with a higher likelihood as T becomes large, and occurs with probability one as $T \to \infty$ conditional on $\theta = L$.

In contrast to the voluntary regime, conditional on $\theta = H$, a *DOWN* cascade cannot occur, regardless of the initial belief. While the *H*-seller never accepts an offer of v_L, rejection of low offers by the seller is informative for the market, allowing market beliefs to improve when enough periods have passed without trade at the low price. As such, even if beliefs are in Case 1, we do not observe *DOWN* cascades conditional on $\theta = H$. Likewise, with respect to *UP* cascades, even if buyers observe that trade occurred at the high price, they continue to update based on their private signals, given that the posterior belief has not reached one.

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[36] The result that the *H*-seller can be trapped in a *DOWN* cascade resembles the "reputation trap" in Levine (2021), wherein a string of bad events destroys the reputation-building efforts of a long-lived agent. In our model, a cascade is driven by the seller's endogenous provision (and withholding) of information, whereas in Levine (2021), information is exogenously given.

This posterior convergence does not occur when $T < \infty$, which implies that UP cascades additionally cannot occur conditional on $\theta = H$ when $T < \infty$. However, as the time horizon becomes infinitely long, market beliefs converge to one that the seller is of type H, as the H-seller never accepts low offers. Hence, conditional on $\theta = H$, an UP cascade occurs with probability one in the mandatory regime as $T \to \infty$.

As in the voluntary case, we define $T_k^{\mathcal{M}}$ to be the minimum number of periods for a $DOWN$ cascade to occur in the mandatory regime when π_0 is Case k (with the value derived in the appendix).

Proposition A.3. *Consider the mandatory regime. Regardless of the initial belief $\pi_0 \in (0,1)$, a cascade occurs only with learning. Moreover, the equilibrium exhibits the following properties:*
  (i) *Suppose $\theta = H$. If $T < \infty$, then neither a DOWN nor UP cascade occurs with positive probability. As $T \to \infty$, an UP cascade occurs with probability one.*
  (ii) *Suppose $\theta = L$. If $T \geq T_k^{\mathcal{M}}$, a DOWN cascade with learning occurs with probability strictly between zero and one. In addition, as $T \to \infty$, a DOWN cascade with learning occurs with probability one.*

Propositions A.2 and A.3 would suggest that mandatory disclosure should be better in a long horizon, as a $DOWN$ cascade without learning can occur with positive probability in the voluntary regime when $\theta = H$, whereas this situation does not arise in the mandatory regime. Moreover, market beliefs converge to the true state in the mandatory regime with probability one when $T \to \infty$, resulting in completely efficient trade, even though this only occurs conditional on $\theta = L$ in the voluntary regime. However, as established in Theorem 1, voluntary disclosure can have welfare advantages over mandatory disclosure when initial beliefs and the L-seller's mimicry incentive are high.

A.3 Unobservable Trading Price

In the baseline model, we assume that the seller can credibly disclose (or is mandated to disclose) the trading price from a past trade. Disclosure therefore conveys two pieces of information: a trade occurred and the price at which it occurred. To highlight the role of the trading price, we consider an extension in which the trading price cannot be credibly disclosed. Specifically, under the voluntary regime, the seller who traded in the previous period decides whether to disclose the occurrence of trade; under the mandatory regime, the seller who traded in the previous period must disclose that trade occurred. As such, upon disclosure, buyers only observe that trade occurred in the previous period, but not the trade price.

A.3.1 Voluntary Disclosure Regime

In the voluntary regime, the absence of trading price information in disclosure allows the L-seller to mimic the disclosure behavior of the H-seller. To see this, consider initial belief $\pi_0 \in (\underline{\pi}, \pi^\dagger)$ (i.e., Case 2 or 3). Suppose that the seller's acceptance strategy is the same as in the baseline model (Proposition 1)—the H-seller only accepts offers that are weakly above c_H, and the L-seller accepts all on-path offers but conceals trade at v_L. Under this

strategy profile, even though the trading price is unobservable, disclosure implies that the offer was $p_1 \geq c_H$. Accordingly, second-period buyers update their beliefs as in the baseline model, with an upward revision to π_1^D following disclosure and a downward revision to π_1^{ND} following non-disclosure. However, under this strategy profile, the L-seller has an incentive to disclose trade that occurred at $p_1 = v_L$, as buyers cannot infer from disclosure that the offer was low and therefore update their beliefs upward to π_1^D. Hence, this strategy profile cannot be an equilibrium when the trading price is unobservable.

The above discussion suggests that when the trading price is unobservable, the L-seller can deviate whenever disclosure constitutes good news. Indeed, we find that this deviation incentive strips disclosure of any informational content.

Proposition A.4. *Consider the voluntary disclosure regime with unobservable trading price. In any PBE, regardless of the initial belief, disclosure reveals no information on the equilibrium path:* $\pi_1^D = \pi_1^{ND} = \pi_0$.

Proof. As in Proposition 1, the θ-seller accepts any offer $p_1 \geq c_\theta$ and rejects any offer $p_1 < c_\theta$. As a result, buyers make offers according to Lemma 1. Since the L-seller always trades in the first period, the L-seller can always choose to disclose (withhold) trade. When $\pi_1^D > \pi_1^{ND}$, the L-seller discloses with probability one; then non-disclosure arises only from the H-seller's rejection, implying that $\pi_1^{ND} = 1$, a contradiction. When $\pi_1^D < \pi_1^{ND}$, the L-seller withholds trade with probability one; then disclosure arises only from the H-seller's acceptance, implying that $\pi_1^D = 1$, a contradiction. Hence, $\pi_1^D \neq \pi_1^{ND}$ cannot hold in equilibrium.

$\square$

This result underscores the importance of trading price information in the baseline model. When trading price information is unobservable, the voluntary disclosure regime collapses to the benchmark no-disclosure (opaque) regime, studied in Sections 3.1 and A.1, in which current buyers receive no information about past trade.

A.3.2 Mandatory Disclosure Regime

We next consider the mandatory regime with unobservable trading prices. Under the mandatory regime, even with unobservable trading prices, the seller's rejection of offers can convey information to future buyers. However, the absence of trading price information weakens the information value of rejection for the H-seller. To see why, consider initial belief $\pi_0 \in (\underline{\pi}, \pi^\dagger)$ (i.e., Case 2 or 3). We construct an equilibrium in which the H-seller accepts any offer above c_H, and the L-seller rejects the low offer v_L with positive probability, as in the baseline model (Proposition 2).

Let ϕ be the probability that the L-seller accepts the v_L-offer. Acceptance (disclosure of trade) implies that either $p_1 \geq c_H$ or that the trade price was v_L and the L-seller accepted this low offer. Accordingly, the belief after acceptance, denoted by π_1^A, is given by

$$\pi_1^A = \frac{\mathbb{P}(p_1 \geq c_H)\mathbb{P}(\theta = H \mid p_1 \geq c_H)}{\mathbb{P}(p_1 \geq c_H) + \mathbb{P}(p_1 = v_L)(1 - \mathbb{P}(\theta = H \mid p_1 = v_L))\phi}. \tag{6}$$

Alternatively, rejection (non-disclosure of trade) implies that both offers were v_L and that both seller types rejected all offers, so the belief after rejection, denoted by π_1^R, is the same as in the baseline model.

When the L-seller is indifferent between accepting and rejecting the v_L-offer (i.e., $\phi \in (0, 1)$), the following condition must hold:[37]

$$c_L + \delta Q_L(\pi_1^R) = v_L + \delta Q_L(\pi_1^A). \tag{7}$$

The left-hand side is the L-seller's expected payoff from rejecting the v_L-offer; the right-hand side from acceptance. The difference from the baseline model (see condition (3)) arises in the belief after acceptance, π_1^A, which is 0 under the baseline model but strictly positive here. This is because acceptance alone does not necessarily imply that the offer was v_L. As such, the L-seller has a weaker incentive to reject the v_L-offer when trading prices are unobservable.

Indeed, condition (7) implies that $\pi_1^R > \pi_1^A$—rejection is always good news. If acceptance were instead good news, then the L-seller would accept the v_L-offer with probability one, which would in turn render acceptance bad news.[38] As a result, rejection still serves as a signal, but its informativeness is capped by unobservable trading prices. By contrast, the baseline model with observable trading prices more sharply isolates the underlying signaling incentives.

A.4   Persistence

In the baseline setting, we assume that the state θ, and thus the quality of the asset or the seller's type, is persistent across periods, which allows us to cleanly illustrate the underlying economic insights. We now relax this assumption and introduce impersistence in the quality of the asset across the two periods. We denote the t-period state as $\theta_t \in \{L, H\}$ for $t = 1, 2$, which is perfectly observed by the seller. As in the baseline model, buyers in period t observe imperfect private signals of the current-period θ_t. The transition probability from each first-period state θ_1 to the low state is denoted as $\lambda_{\theta_1} \in (0, 1)$, i.e., $\mathbb{P}(\theta_2 = L \mid \theta_1 = L) = \lambda_L$ and analogously $\mathbb{P}(\theta_2 = L \mid \theta_1 = H) = \lambda_H$ (transition from θ_1 to the high state is thus given as $1 - \lambda_{\theta_1}$).[39] We focus on the case where asset quality is positively serially correlated in time, i.e., $\lambda_L \geq \lambda_H$.

In this extended setting, equilibrium features are quantitatively similar to the baseline setting. To see why, consider the voluntary regime. Denote by $\pi_1^d := \mathbb{P}(\theta_1 = H \mid d)$ the posterior belief about the first-period state upon disclosure outcome $d \in \{D, ND\}$ of the first-period trade. Similarly, denote by $\tilde{\pi}_1^d := \mathbb{P}(\theta_2 = H \mid d)$ the posterior belief about the

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[37]Following similar arguments as in the proof of Proposition 2, we can show that an equilibrium always exists (with a first-period bidding adjustment ξ).

[38]More formally, if $\pi_1^A \geq \pi_1^R$, then the right-hand side of (7) is strictly greater than the left-hand side, which implies that the L-seller accepts the v_L-offer with probability one. But then $\pi_1^R = 1$, and Bayes' law implies $\pi_1^A < \pi_0$, a contradiction.

[39]To avoid additional signaling incentives, we assume that the transition of types occurs after the disclosure of past trade, if the seller chooses to disclose, in the second period.

second-period state. Then, by Bayes' law, we have

$$\tilde{\pi}_1^d = \pi_1^d(1 - \lambda_H) + (1 - \pi_1^d)(1 - \lambda_L).$$

Since $\lambda_L \geq \lambda_H$, we have $\tilde{\pi}_1^D > \tilde{\pi}_1^{ND}$: disclosure continues to result in upward updating of beliefs by second-period buyers, while non-disclosure results in downward updating. Therefore, the voluntary disclosure equilibrium takes the same form as in the baseline model. We can also verify that the mandatory disclosure regime exhibits similar equilibrium features.

Importantly, under both regimes, the belief updating is continuous in the degree of persistence. As such, when the degree of persistence is sufficiently high (i.e., λ_L is sufficiently close to 1 and λ_H is sufficiently close to 0), welfare comparisons between the two regimes continue to hold as in the baseline model. Thus our main result (Theorem 1) is robust to the introduction of a small degree of impersistence.

A.5 Stochastic buyer arrival

In our baseline model, the only possibility of trade not occurring is due to the seller rejecting offers. While this is reasonable in most situations, there may be factors outside of the seller's control that prevent trade from occurring. We now consider search friction—the possibility that trade cannot happen due to, for example, buyers failing to arrive in the market. Specifically, we assume with probability $s \in (0, 1)$ that buyers arrive in the first period and observe private signals. For parsimony, buyers continue to arrive with probability one in the second period.

As in the case of imperfect persistence, we find that the main features of the equilibrium are preserved in this extended setting. In particular, under both regimes, the belief updating after a disclosure outcome is given by a weighted average of the posterior belief under no search friction ($s = 1$) and the prior belief. For example, under the voluntary regime, the posterior belief after non-disclosure, denoted by $\tilde{\pi}_1^{ND}$, is

$$\tilde{\pi}_1^{ND} = \mathbb{P}(\text{arrival} \mid ND)\, \pi_1^{ND} + (1 - \mathbb{P}(\text{arrival} \mid ND))\, \pi_0, \tag{8}$$

where π_1^{ND} is the posterior belief in the baseline model without search friction. The posterior belief of buyer arrival given non-disclosure is

$$\mathbb{P}(\text{arrival} \mid ND) = \frac{s\mathbb{P}(p_1 = p_{\text{low}})}{s\mathbb{P}(p_1 = p_{\text{low}}) + (1 - s)}.$$

In the event that no buyers arrive in the first period, second-period buyers simply retain the prior belief π_0; otherwise, they update their beliefs as in the baseline model. The expression (8) shows that the belief updating is the weighted average of these two cases. In contrast, when the seller discloses trade, the second-period buyers learn that the first-period buyers arrived, so the belief updating remains unchanged from the baseline model: $\tilde{\pi}_1^D = \pi_1^D$.

Since the belief updating is again continuous in the degree of search friction, the qualitative features of the equilibrium, including the welfare comparison between the two regimes, are preserved when search friction is sufficiently small (i.e., s is sufficiently close to one).

B Proofs

B.1 List of Notations

We collect the list of notations used in the proof.

Static Bidding Game

$\pi_0 \in (0,1)$	The prior belief in the first period
$\pi_{t-1} \in (0,1)$	The *initial belief* in period t: the public belief in period t before the bidding stage (but after the disclosure stage in period t)
$V(\pi) = \mathbb{E}[v_\theta; \pi] = \pi v_H + (1-\pi)v_L$	The expected value of the asset at belief π.
$x_{i,t}$	Buyer i's bid in period t
$p_t = \max\{x_{i,t}, x_{j,t}\}$	The offer in period t (i.e., the highest bid in period t)
$p_{\text{high}}, p_{\text{low}}$	For Cases 1–3 of Lemma 1, p_{high} is an offer weakly greater than c_H, and $p_{\text{low}} = v_L$. For Case 4, $p_{\text{high}} > V(\pi^{\ell\ell})$ and $p_{\text{low}} = V(\pi^{\ell\ell}) \geq c_H$.
$u_{B,s_i}[x]$	Buyer i's expected payoff when he bids x.
$z_t p_t + (1-z_t)c_\theta$	θ-seller's period-payoff given realized trade $z_t \in \{0,1\}$ and price p_t
$u_{S,\theta}(\pi) = \mathbb{E}_{(z_t,p_t)}[z_t p_t + (1-z_t)c_\theta \mid \theta; \pi]$	θ-seller's expected equilibrium payoff in period t given bidding-stage belief π
$u_{B,(s_i,s_j)}(\pi)$	Buyer i's expected payoff when the signal realization is (s_i, s_j).
$u_{B,s_i}(\pi_{t-1}) = \sum_{s_j} \mathbb{P}(s_j \mid s_i; \pi_{t-1})u_{B,(s_i,s_j)}(\pi_{t-1})$	Buyer i's expected payoff given signal s_i and prior π.
$\pi_t^{s_i} = \mathbb{P}(\theta = H \mid s_i; \pi_{t-1})$ $\pi_t^{s_i s_j} = \mathbb{P}(\theta = H \mid s_i, s_j; \pi_{t-1})$	The posterior beliefs after signal realizations given the initial belief π_{t-1}
$\underline{\pi}, \overline{\pi}, \pi^\dagger$	See Lemma 1. Case 1: $\pi_0 \leq \underline{\pi}$, Case 2: $\pi_0 \in (\underline{\pi}, \overline{\pi}]$, Case 3: $\pi_0 \in (\overline{\pi}, \pi^\dagger)$, Case 4: $\pi_0 \geq \pi^\dagger$
$\pi^* \in (\overline{\pi}, \pi^\dagger)$	The belief cutoff in Theorem 1. See Lemma B.6.

Voluntary Regime

$d_{t-1} \in \{ND, D\}$	The disclosure decision for the period-$t-1$ trade. Note that disclosure d_{t-1} occurs at the start of period t. In the two-period model, we drop the time subscript.
$\pi_{t-1} = \mathbb{P}(\theta = H \mid h^{t-1})$ or $\pi_{t-1}^{d_{t-1}} = \mathbb{P}(\theta = H \mid h^{t-2}, d_{t-1})$	The "initial belief" in period t after the disclosure stage. The latter expression writes the disclosure outcome of period-$(t-1)$ trade explicitly.

Mandatory Regime

$a_{t-1} \in \{R, p_{\text{high}}, p_{\text{low}}\}$	The acceptance outcome for the period-$(t-1)$ trade under the mandatory regime, with R indicating rejection, p_{high} acceptance of a high offer, and p_{low} acceptance of a low offer
$\pi_{t-1} = \mathbb{P}(\theta = H \mid h^{t-1})$ or $\pi_{t-1}^{a_{t-1}} = \mathbb{P}(\theta = H \mid h^{t-2}, a_{t-1})$	The "initial belief" in period t after the disclosure stage. The latter expression writes the disclosure/acceptance outcome of period-$(t-1)$ trade explicitly. For Cases 1–3, $\pi_{t-1}^{p_{\text{low}}} = 0$ and $\pi_{t-1}^{p_{\text{high}}} = \pi_{t-1}^D$, where π_{t-1}^D is the same as in the voluntary regime. For Case 4, $\pi_{t-1}^{p_{\text{low}}} = \pi_{t-1}^{ND}$ and $\pi_{t-1}^{p_{\text{high}}} = \pi_{t-1}^D$.
$\xi \geq 0$	The parameter that characterizes the first-period bidding equilibrium in the mandatory regime. The bidders and the H-seller behave as if the game is static and the H-seller's reservation value is $\tilde{c}_H := c_H + \xi$.
$\mathbb{P}_\xi(p_t \in \cdot)$	The probability distribution of the period-t offer p_t when the bidding game is the ξ-static bidding equilibrium (i.e., the buyers bid as if c_H is replaced by $\tilde{c}_H = c_H + \xi$).

Welfare

$\mathcal{R} \in \{\mathcal{D}, \mathcal{M}, \mathcal{O}\}$	Disclosure regime: $\mathcal{D}$ is the voluntary disclosure regime, $\mathcal{M}$ is the mandatory disclosure regime, and $\mathcal{O}$ is the opaque regime.
$u_B(\pi_{t-1}) = \sum_{s_i, s_j} \mathbb{P}(s_i, s_j; \pi_{t-1}) u_{B, (s_i, s_j)}(\pi_{t-1})$ $= \sum_{s_i} \mathbb{P}(s_i; \pi_{t-1}) u_{B, s_i}(\pi_{t-1})$	Buyer i's expected payoff given signal prior π.
$U_B^{\mathcal{R}}(\pi_0) = \sum_{t=1}^T \delta^{t-1} \mathbb{E}[2u_B(\pi_{t-1}) \mid \pi_0]$	Total buyers' discounted sum of expected payoffs in regime $\mathcal{R}$.
$U_{S,\theta}^{\mathcal{R}}(\pi_0) = \mathbb{E}[\sum_{t=1}^T \delta^{t-1} u_{S,\theta}(\pi_{t-1}) \mid \pi_0]$	The θ-seller's discounted sum of expected payoffs in regime $\mathcal{R}$.

$Q_\theta^{(T-t)}(\pi_{t-1}) := \mathbb{E}[\sum_{\tau=t}^{T-1} \delta^{\tau-t} u_{S,\theta}(\pi_{\tau-1}); \pi_{t-1}]$	The θ-seller's continuation payoff from period t given the initial belief π_{t-1}.
$w_\theta(\pi_{t-1}) = \mathbb{E}[z_t v_\theta + (1-z_t)c_\theta \mid \theta; \pi_{t-1}]$	Expected welfare in state θ in period t given π_{t-1}.
$W_\theta^{\mathcal{R}}(\pi_0) = \sum_{t=1}^{T} \delta^{t-1} \mathbb{E}[w_\theta(\pi_{t-1}) \mid \pi_0]$	Discounted sum of expected welfare given θ.
$W^{\mathcal{R}}(\pi_0) = \pi_0 W_H^{\mathcal{R}}(\pi_0) + (1-\pi_0)W_L^{\mathcal{R}}(\pi_0)$	Discounted sum of expected welfare.

B.2 Proof of Lemma 1

In the proof below, we use notation $V(\pi) = \pi v_H + (1-\pi)v_L = \mathbb{E}[v_\theta; \pi]$ to express the expected value of the asset given belief π.

Proof. Let $\pi^{s_i} := \mathbb{P}(\theta = H \mid s_i; \pi)$ and $\pi^{s_i s_j} := \mathbb{P}(\theta = H \mid s_i, s_j; \pi)$ denote the posterior beliefs after observing the signals. Let $\mathbb{P}(s_i \mid s_j; \pi)$ be the probability that buyer i observes s_i conditional on buyer j observing s_j, given the prior π. The conditional expectations are now written as $V(\pi^{s_i}) := \mathbb{E}[v_\theta \mid s_i; \pi]$ and $V(\pi^{s_i s_j}) := \mathbb{E}[v_\theta \mid s_i, s_j; \pi]$.

The cases $\pi \in \{0,1\}$ are trivial—they are the standard Bertrand competition. For $\pi \in (0,1)$, we verify that the equilibrium bidding strategy described in the main text is indeed the equilibrium.

Let $u_{B,s_i}[x_i]$ be the expected payoff of an s_i-buyer when he bids x_i.

Case 1. (Low beliefs.) $V(\pi^h) \leq c_H$. Suppose that $c_H \geq V(\pi^{hh})$. The h-buyer finds it unprofitable to target the H-seller even when the other buyer receives a high signal. Thus, both buyer types bid v_L. Now suppose that $c_H \in [V(\pi^h), V(\pi^{hh}))$. Clearly, the ℓ-buyer optimally targets only the L-seller. Consider next an h-buyer. The best deviation is to overbid c_H,[40] which gives a payoff of

$$\begin{aligned} u_{B,h}[c_H] &= \mathbb{P}(\ell \mid h)(V(\pi) - c_H) + \mathbb{P}(h \mid h)(V(\pi^{hh}) - c_H) \\ &\leq \mathbb{P}(\ell \mid h)(V(\pi) - V(\pi^h)) + \mathbb{P}(h \mid h)(V(\pi^{hh}) - V(\pi^h)) \\ &= 0. \end{aligned}$$

Hence, this deviation is unprofitable.

Case 2. (Low-intermediate beliefs.) $V(\pi) \leq c_H < V(\pi^h)$. Define

$$\sigma = \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{c_H - V(\pi)}{V(\pi^{hh}) - c_H}, \quad \tilde{G}_2(x) = \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{(x - V(\pi))}{(V(\pi^{hh}) - x)}.$$

Consider an ℓ-buyer. He expects zero profit by making an offer of v_L. If he deviates to $x < v_L$, he gets zero profit as well. If he deviates to $x \in (v_L, c_H)$, his expected payoff is $u_{B,\ell}[x] = \mathbb{P}(\ell \mid \ell)(v_L - x) + \mathbb{P}(h \mid \ell)\sigma(v_L - x) < 0$. Since $c_H \geq V(\pi)$, bidding $x \geq c_H$ is also not profitable. Consider next an h-buyer. If

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[40]We say that a buyer overbids x if he bids $x + \varepsilon$, where $\varepsilon > 0$ is an arbitrarily small number. For brevity, we omit ε from the payoff computation.

he overbids $V(\pi^h) = \mathbb{P}(\ell \mid h)V(\pi) + \mathbb{P}(h \mid h)V(\pi^{hh})$, his expected payoff is

$$\mathbb{P}(\ell \mid h)(V(\pi) - V(\pi^h)) + \mathbb{P}(h \mid h)(V(\pi^{hh}) - V(\pi^h)) = 0.$$

If he overbids v_L, he receives zero profit. The indifference condition combined with $\tilde{G}_2(c_H) = \sigma$ gives $\tilde{G}_2$ and σ.

Case 3. (High-intermediate beliefs.) $V(\pi^{\ell\ell}) < c_H < V(\pi)$. Define

$$G_3(x) = \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{(x - c_H)}{(V(\pi^{hh}) - x)}.$$

Consider an h-buyer. If he overbids $x = \mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi^{hh})$, his expected payoff is $\mathbb{P}(\ell \mid h)(V(\pi) - x) + \mathbb{P}(h \mid h)(V(\pi^{hh}) - x) = \mathbb{P}(\ell \mid h)(V(\pi) - c_H) > 0$. If he overbids c_H, he again expects $\mathbb{P}(\ell \mid h)(V(\pi) - c_H) > 0$. He cannot do better than this. The indifference condition determines G_3. Consider next an ℓ-buyer. If he bids v_L, he expects zero profit. If he bids $x \in [c_H, \mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi^{hh})]$, he expects

$$u_{B,\ell}[x] = \mathbb{P}(\ell \mid \ell)(V(\pi^{\ell\ell}) - x) + \mathbb{P}(h \mid \ell)G_3(x)(V(\pi) - x).$$

The first term is always negative, and the second term is negative for $x > V(\pi)$. Suppose next that $x \in (c_H, V(\pi))$. Observe that

$$\frac{\partial u_{B,\ell}[x]}{\partial x} = -\mathbb{P}(\ell \mid \ell) + \mathbb{P}(h \mid \ell)\frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)}\left[\frac{V(\pi^{hh}) - c_H}{(V(\pi^{hh}) - x)^2}(V(\pi) - x) - \frac{x - c_H}{V(\pi^{hh}) - x}\right]$$

and

$$\frac{\partial^2 u_{B,\ell}[x]}{\partial x^2} = -\mathbb{P}(h \mid \ell)\frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)}\frac{2(V(\pi^{hh}) - V(\pi))(V(\pi^{hh}) - c_H)}{(V(\pi^{hh}) - x)^3} < 0.$$

Since $\frac{\partial u_{B,\ell}[c_H]}{\partial x} < 0$, we have that $\frac{\partial u_{B,\ell}[x]}{\partial x} < 0$ for all $x \in (c_H, V(\pi))$. Since

$$u_{B,\ell}[c_H] = \mathbb{P}(\ell \mid \ell)(V(\pi^{\ell\ell}) - c_H) < 0,$$

the deviation is not profitable. Finally, if he overbids $\mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi^{hh})$, he expects

$$\mathbb{P}(\ell \mid \ell)V(\pi^{\ell\ell}) + \mathbb{P}(h \mid \ell)V(\pi) - (\mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi^{hh}))$$
$$< \mathbb{P}(\ell \mid \ell)V(\pi^{\ell\ell}) + \mathbb{P}(h \mid \ell)V(\pi) - (\mathbb{P}(\ell \mid h)c_H + \mathbb{P}(h \mid h)V(\pi))$$
$$< 0,$$

where the last inequality is from $\mathbb{P}(h|h) - \mathbb{P}(h \mid \ell) = (\pi^h - \pi^\ell)(2q - 1) > 0$. Hence, the deviation is not profitable.

Case 4. (High beliefs.) $V(\pi^{\ell\ell}) > c_H$. Define

$$G_4(x) = \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{(x - V(\pi^{\ell\ell}))}{(V(\pi^{hh}) - x)}.$$

By overbidding $V(\pi^{\ell\ell})$, the h-bidder can secure $\mathbb{P}(\ell \mid h)(V(\pi) - V(\pi^{\ell\ell}))$. By bidding in the support of G_4, he expects $u_{B,h}[x] = \mathbb{P}(\ell \mid h)(V(\pi) - x) + \mathbb{P}(h \mid h)G_4(x)(V(\pi^{hh}) - x)$. The indifference condition determines G_4. Consider an ℓ-buyer. He expects zero profit by bidding $V(\pi^{\ell\ell})$. As in the previous case, one can show that bidding $x > V(\pi^{\ell\ell})$ is not profitable.

$\square$

From the above, we can calculate the expected payoffs of each agent. We summarize the results in Table 2. In the table, we express the thresholds of cases in terms of prior belief π by $\underline{\pi}$, $\overline{\pi}$, and $\pi^{\dagger}$, which are defined as follows:

$$\underline{\pi} := \frac{\overline{\pi}(1-q)}{q - \overline{\pi}(2q-1)}, \quad \overline{\pi} := \frac{c_H - v_L}{v_H - v_L}, \quad \pi^{\dagger} := \frac{q^2\overline{\pi}}{(1-q)^2 + (2q-1)\overline{\pi}}.$$

Each threshold is determined so that $\underline{\pi}^h = \overline{\pi}$, $V(\overline{\pi}) = c_H$, and $(\pi^{\dagger})^{\ell\ell} = \overline{\pi}$.

Table 2: Expected Payoffs in the Static Bidding Game

π	$u_{B,\ell}(\pi)$	$u_{B,h}(\pi)$	$u_{S,L}(\pi)$	$u_{S,H}(\pi)$
Case 1: $\pi \leq \underline{\pi}$	0	0	v_L	c_H
Case 2: $\pi \in (\underline{\pi}, \overline{\pi}]$	0	0	$\mathbb{P}(p = v_L \mid L)v_L +$ $\mathbb{P}(p \geq c_H \mid L)\mathbb{E}[p \mid p \geq c_H]$	$\mathbb{P}(p = v_L \mid H)c_H +$ $\mathbb{P}(p \geq c_H \mid H)\mathbb{E}[p \mid p \geq c_H]$
Case 3: $\pi \in (\overline{\pi}, \pi^{\dagger})$	0	$\mathbb{P}(\ell \mid h)(V(\pi) - V(\overline{\pi}))$	$\mathbb{P}(p = v_L \mid L)v_L +$ $\mathbb{P}(p \geq c_H \mid L)\mathbb{E}[p \mid p \geq c_H]$	$\mathbb{P}(p = v_L \mid H)c_H +$ $\mathbb{P}(p \geq c_H \mid H)\mathbb{E}[p \mid p \geq c_H]$
Case 4: $\pi \geq \pi^{\dagger}$	0	$\mathbb{P}(\ell \mid h)(V(\pi) - V(\pi^{\ell\ell}))$		$\mathbb{P}(p = V(\pi^{\ell\ell}) \mid \theta)V(\pi^{\ell\ell}) + \mathbb{P}(p \geq V(\pi^{\ell\ell}) \mid \theta)\mathbb{E}[p \mid p \geq V(\pi^{\ell\ell})]$

Note: This table shows the expected payoffs in the static bidding game. The offer p denotes the maximum of two bids.

Figure 1 shows the expected payoffs of the seller in the static bidding equilibrium.

B.3 Proof of Proposition 1

Lemma B.1. *In any PBE of the voluntary disclosure game, the θ-seller accepts any first-period offer $p_1 \geq c_\theta$.*

Proof. The second-period buyers' beliefs (and thus the seller's continuation payoffs) depend only on whether the trade in the first period is disclosed or not. Therefore, accepting

Figure 1: Expected payoffs of the seller in the static bidding game
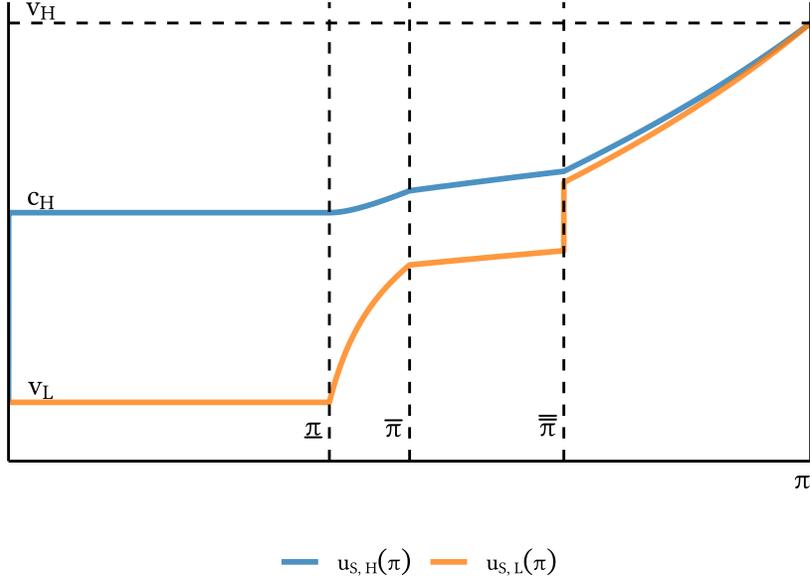
Note: The figure shows the expected payoffs of the seller for each initial level of belief π according to Lemma 1. The L-seller's expected payoff is discontinuous at $\pi = \pi^\dagger$, and we plot the line segment connecting the left and right limits. Any point in the line segment can realize depending on the buyer's mixed strategy. See Lemma B.3 for more on this point. The parameters we used to draw the figure are as follows: $q = 0.6, \pi_0 = 0.65, v_H = 5, v_L = 3, c_H = 4$.

$p_1 \geq c_\theta$ and withholding yields the same public history as rejecting but strictly higher current payoff. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Proof of Proposition 1. First, we consider Cases 2–3. We examine deviation incentives from the prescribed strategy profile. Since the θ-seller accepts any offers $p_1 \geq c_\theta$, buyers have no incentive to deviate, as shown in Lemma 1. Under the proposed equilibrium strategy, the updated belief after disclosure of a high offer p_1 is given by

$$\pi_1^D = \mathbb{P}(\theta = H \mid p_1 \geq c_H; \pi_0)$$
$$= \frac{2\mathbb{P}(s_i = h, s_j = \ell; \pi_0)(1 - \sigma)\pi_0 + \mathbb{P}(s_i = h, s_j = h; \pi_0)(1 - \sigma^2)\pi_1^{hh}}{2\mathbb{P}(s_i = h, s_j = \ell; \pi_0)(1 - \sigma) + \mathbb{P}(s_i = h, s_j = h; \pi_0)(1 - \sigma^2)}$$
$$> \pi_0.$$

We use the convention that σ, the probability of h-bidder bidding v_L, is 0 in Case 3. By Bayes' law, we have $\pi_1^{ND} < \pi_0$.

Let $u_{S,\theta}(\pi)$ be the θ-seller's expected payoff in the static bidding equilibrium of Lemma 1. It is the θ-seller's continuation payoff given the continuation belief π. Consider the H-seller. For any offer $p_1 \geq c_H$, she obtains the expected payoff of $p_1 + \delta u_{S,H}(\pi_1^D)$ by accepting the offer and disclosing it. If the seller accepts and conceals the trade at a price $p_1 \geq c_H$, her

expected payoff is $p_1 + \delta u_{S,H}(\pi_1^{ND})$. Since $u_{S,H}$ is weakly increasing, this deviation is not profitable. Alternatively, for any offer $p_1 < c_H$, she obtains $c_H + \delta u_{S,H}(\pi_1^{ND})$ by rejecting and concealing the trade in the proposed equilibrium. Since $p_1 < c_H$, deviating to accepting and concealing the trade is clearly not profitable. If she deviates by accepting the offer and disclosing it, her expected payoff is $p_1 + \delta u_{S,H}(\pi^o)$, where π^o is the buyer's off-path belief given disclosure of $p_1 < c_H$. Since $p_1 < c_H$, any $\pi^o \leq \pi_1^{ND}$ deters this deviation.

Next, consider the L-seller. For any offer $p_1 < c_H$, the L-seller obtains the expected payoff of $p_1 + \delta u_{S,L}(\pi_1^{ND})$ by accepting the offer and concealing it in the proposed equilibrium. Since $p_1 > c_L$, the best deviation is to accept the offer and disclose it, yielding the expected payoff of $p_1 + \delta u_{S,L}(\pi^o)$. Thus any off-path belief $\pi^o \leq \pi_1^{ND}$ deters the deviation.

For Case 4, both sellers accept all on-path offers. Any offers $p_1 > V(\pi_1^{\ell\ell})$ are disclosed, because disclosure indicates that at least one of the signals in the previous period was high, leading to upward belief revision. The disclosure of an offer $p_1 = V(\pi_1^{\ell\ell})$ reveals that the first-period buyers' signals were (ℓ, ℓ). Since both types accept this offer, the seller is indifferent between disclosing and concealing it. In either case, buyers update their belief to $\pi_1^{ND} = \pi_1^{\ell\ell}$. For the sake of exposition, we assume that the seller does not disclose trade at $V(\pi_1^{\ell\ell})$.

For Case 1, the only on-path offer is $p_1 = v_L$. In the candidate equilibrium, the H-seller rejects the offer v_L and the L-seller accepts and withholds it. There is no belief updating from the disclosure stage, so $\pi_1^D = \pi_1^{ND} = \pi_0$. Any off-path belief lower than the prior supports this equilibrium. $\quad\square$

We summarize the belief updating in the voluntary regime in Table 3.

Table 3: Beliefs in the Voluntary Regime

π_0	π_1^D	π_1^{ND}
Case 1: $\pi \leq \underline{\pi}$	π_0	π_0
Case 2: $\pi \in (\underline{\pi}, \overline{\pi}]$	$\mathbb{P}(\theta = H \mid p_1 \geq c_H)$	$\mathbb{P}(\theta = H \mid p_1 = v_L)$
Case 3: $\pi \in (\overline{\pi}, \pi^\dagger)$	$\mathbb{P}(\theta = H \mid (s_i, s_j) \in \{(\ell, h), (h, \ell), (h, h)\})$	$\pi_1^{\ell\ell}$
Case 4: $\pi \geq \pi^\dagger$	$\mathbb{P}(\theta = H \mid (s_i, s_j) \in \{(\ell, h), (h, \ell), (h, h)\})$	$\pi_1^{\ell\ell}$

Note: This table shows the belief updating in the voluntary regime. The belief π_1^D is the posterior belief upon disclosure, and π_1^{ND} is the posterior belief upon non-disclosure. In Case 2, because of the winner's curse, the belief updating depends on σ.

For Cases 3 and 4, the supports of h-buyers' bids and ℓ-buyers' bids are distinct. Therefore, disclosure corresponds to the straightforward partition of the signal space. For Case 2, because of the winner's curse, h-bidders bid v_L with probability σ. Therefore, the belief updating is more complicated. The following expressions give the explicit forms of the updated

beliefs:

$$\mathbb{P}(\theta = H \mid (s_i, s_j) \in \{(\ell, h), (h, \ell), (h, h)\}) = \frac{2q(1-q)\pi_0 + \{\pi_0 q^2 + (1-\pi_0)(1-q)^2\}\pi_1^{hh}}{2q(1-q) + \{\pi_0 q^2 + (1-\pi_0)(1-q)^2\}}$$

and

$$\mathbb{P}(\theta = H \mid p_1 \geq c_H) \tag{9}$$
$$= \frac{2q(1-q)(1-\sigma)\pi_0 + \{\pi_0 q^2 + (1-\pi_0)(1-q)^2\}(1-\sigma^2)\pi_1^{hh}}{2q(1-q)(1-\sigma) + \{\pi_0 q^2 + (1-\pi_0)(1-q)^2\}(1-\sigma^2)}$$
$$\mathbb{P}(\theta = H \mid p_1 = v_L)$$
$$= \frac{\{\pi_0(1-q)^2 + (1-\pi_0)q^2\}\pi_1^{\ell\ell} + 2q(1-q)\sigma\pi_0 + \{\pi_0 q^2 + (1-\pi_0)(1-q)^2\}\sigma^2\pi_1^{hh}}{\{\pi_0(1-q)^2 + (1-\pi_0)q^2\} + 2q(1-q)\sigma + \{\pi_0 q^2 + (1-\pi_0)(1-q)^2\}\sigma^2}$$

We later use the following property:

Lemma B.2. *Suppose that π_0 is Case 2. Then, π_1^D is increasing in c_H.*

Proof. Rewriting (9) yields

$$\pi_1^D = \frac{2q(1-q) + q^2(1+\sigma)}{2q(1-q) + \{\pi_0 q^2 + (1-\pi_0)(1-q)^2\}(1+\sigma)}\pi_0.$$

Differentiating this with respect to σ implies that π_1^D is increasing in σ. Furthermore, from the definition of σ in Lemma 1, we have

$$\frac{\partial \sigma}{\partial c_H} = \frac{\partial \sigma}{\partial \overline{\pi}} \frac{\partial \overline{\pi}}{\partial c_H} = \frac{\mathbb{P}(\ell \mid h)}{\mathbb{P}(h \mid h)} \frac{(v_H - v_L)(V(\pi_1^{hh}) - V(\pi))}{(V(\pi_1^{hh}) - V(\overline{\pi}))^2} \frac{\partial \overline{\pi}}{\partial c_H} > 0.$$

Thus, π_1^D is increasing in c_H. $\qquad\square$

B.4   Proof of Proposition 2

When $\pi_0 \geq \pi^\dagger$ (Case 4), buyers bid according to Lemma 1 and the seller accepts all on-path offers. Acceptance at $p_{\text{high}} > V(\pi_1^{\ell\ell})$ results in an upward revision of belief to π_1^D while acceptance at $p_{\text{low}} = V(\pi_1^{\ell\ell})$ leads to a downward revision π_1^{ND}, as in the voluntary regime. Therefore, in the remaining part of the proof, we focus on the case $\pi_0 < \pi^\dagger$ (Cases 1–3). A high offer refers to an offer $p_1 \geq c_H$, and a low offer refers to $p_1 = v_L$.

We proceed as follows. First, we suppose (and later show) that the H-seller accepts any on-path offers $p_1 \geq c_H$ and rejects $p_1 = v_L$.[41] As discussed in the main text, the L-seller sometimes rejects the offer to mimic the H-seller's behavior. We prove the existence of a unique rejection belief that solves the L-seller's indifference condition. Second, we derive

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[41]An alternative "pooling" equilibrium may exist, in which the H-seller accepts the v_L-offer to avoid harsh off-path beliefs. In Appendix C, we show that such an unnatural equilibrium is eliminated by the standard refinement.

the first-period bidding equilibrium. We show that the first-period bidding equilibrium is adjusted from the static equilibrium to ensure the H-seller's incentive to accept a high offer with probability one.

Denote by $Q_\theta(\pi) := u_{S,\theta}(\pi)$ the θ-seller's continuation payoff. We introduce the notation Q_θ to be consistent with the long-horizon case.

B.4.1 Deriving the Rejection Belief

Denote by π_1^R the posterior belief upon no trade (rejection) under the mandatory regime. Suppose that the L-seller received an offer of v_L. Instead of directly analyzing the acceptance probability ϕ, we work with the posterior belief upon rejection, π_1^R as the primitive. Once we obtain π_1^R, we can back out the acceptance probability.

We first show that there is a unique rejection belief that solves the L-seller's indifference condition: $(1+\delta)v_L = c_L + \delta Q_L$. Since Q_L is discontinuous at $\pi^\dagger$, when $(1+\delta)v_L - c_L \in \delta(Q_L^-(\pi^\dagger), Q_L^+(\pi^\dagger))$ with Q_L^- and Q_L^+ denoting the left and right limits, we need to specify the second-period buyers' mixing probability to pin down the L-seller's continuation payoff. Let σ_ℓ be the probability that the second-period ℓ-buyer bids v_L (as opposed to c_H).[42] Then, the L-seller's continuation payoff at $\pi^\dagger$ is given by

$$Q_L(\pi^\dagger; \sigma_\ell) = \sigma_\ell^2 Q_L^-(\pi^\dagger) + (1 - \sigma_\ell^2)Q_L^+(\pi^\dagger),$$

where $Q_L(\pi; \sigma_\ell)$ is the L-seller's continuation payoff when the second-period ℓ-buyer bids v_L with probability σ_ℓ at $\pi = \pi^\dagger$.

Lemma B.3. *There exists a unique* $(\hat{\pi}_1^R, \sigma_\ell) \in (0,1) \times [0,1]$ *such that*

$$\delta Q_L(\hat{\pi}_1^R; \sigma_\ell) = (1+\delta)v_L - c_L. \tag{10}$$

Moreover, $\hat{\pi}_1^R > \underline{\pi}$.

Proof. From Lemma 1, $Q_L(\pi; \sigma_\ell)$ is constant on $\pi \in [0, \underline{\pi}]$ and strictly increasing on $\pi \in (\underline{\pi}, 1)$ for any σ_ℓ. Moreover, $Q_L(\pi; \sigma_\ell) = v_L$ on $[0, \underline{\pi}]$ and $Q_L(1; \sigma_\ell) = v_H$. Since any $\sigma_\ell \in [0,1]$ is possible, the mapping $[0,1] \ni \sigma_\ell \mapsto Q_L(\pi^\dagger; \sigma_\ell) \in [Q_L^-(\pi^\dagger), Q_L^+(\pi^\dagger)]$ is bijective.

Since $0 < c_L < v_L$ and $(1+\delta)v_L < c_L + \delta v_H$, we have $(1+\delta)v_L - c_L \in (\delta v_L, \delta v_H)$. Hence, there exists a unique pair $(\hat{\pi}_1^R, \sigma_\ell)$ such that $\delta Q_L(\hat{\pi}_1^R; \sigma_\ell) = (1+\delta)v_L - c_L$, where $\hat{\pi}_1^R > \underline{\pi}$ $\square$

Graphically, the solution to (10) is the intersection of the horizontal line $(1+\delta)v_L - c_L$ and the continuation curve $\delta Q_L(\pi; \sigma_\ell)$. Figure 2 illustrates this for three different levels of c_L. When $c_L = c_L^{medium}$, the L-seller's continuation payoff is in the discontinuous region, so we need to choose the buyers' randomization probability σ_ℓ to ensure the L-seller's indifference.

In Lemma B.3, we do not restrict the solution $\hat{\pi}_1^R$. In equilibrium, the rejection belief should be derived by the Bayesian updating given the seller's equilibrium strategy. The following result establishes the relationship between $\hat{\pi}_1^R$ and the equilibrium rejection belief π_1^R. Our claim is as follows:

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[42]Since the ℓ-bidders are indifferent between bidding v_L and c_H, any σ_ℓ supports the bidding equilibrium.

Figure 2: The L-seller's indifference condition
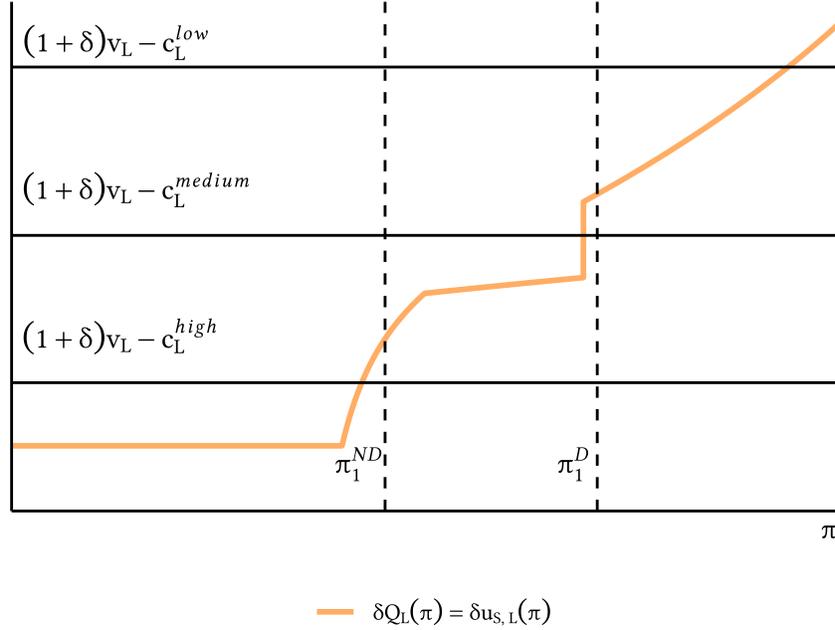
Note: The figure illustrates the seller's indifference condition (10). It plots the seller's continuation value $\delta Q_L(\pi) = \delta u_{S,L}(\pi)$ and the net benefit of accepting the low offer, $(1+\delta)v_L - c_L$, for three different levels of c_L. The intersections of the horizontal lines and the continuation function determine $\hat{\pi}^R$. The parameters we used to draw the figure are as follows: $q = 0.6, \pi_0 = 0.65, v_H = 5, v_L = 3, c_H = 4$.

Lemma B.4. *Let $\hat{\pi}_1^R$ be the solution to* (10). *Then,*

$$\pi_1^R = \max\{\hat{\pi}_1^R, \pi_1^{ND}\},$$

where the belief π_1^{ND} is from the voluntary regime (see Table 3).

Proof. The belief updating after rejection is given by $\pi_1^R = (\pi_1^{ND})/(\pi_1^{ND} + (1-\pi_1^{ND})(1-\phi))$, where $\phi \in [0,1)$ is the probability that the L-seller accepts the v_L-offer. When $\hat{\pi}_1^R \geq \pi_1^{ND}$, there exists an acceptance probability ϕ such that the rejection belief satisfies $\pi_1^R = \hat{\pi}_1^R$. At this belief, the L-seller's indifference condition holds with equality by construction. Alternatively, when $\hat{\pi}_1^R < \pi_1^{ND}$, the L-seller strictly prefers to reject the low offer, implying an acceptance probability of $\phi = 0$; thus $\pi_1^R = \pi_1^{ND}$. $\square$

The following lemma establishes a part of Proposition 2 by showing how the rejection belief compares with the prior belief.

Lemma B.5. *The rejection belief π_1^R satisfies the following:*
- $\pi_1^R < \pi_0$ *if* $(1+\delta)v_L < c_L + \delta Q_L(\pi_0)$;
- $\pi_1^R = \hat{\pi}_1^R$ *if* $(1+\delta)v_L \geq c_L + \delta Q_L(\pi_0)$.

Proof. Suppose that $(1+\delta)v_L < c_L + \delta Q_L(\pi_0)$. The monotonicity of Q_L implies that $\hat{\pi}_1^R < \pi_0$. Since $\pi_1^{ND} < \pi_0$, by Lemma B.4, we have $\pi_1^R < \pi_0$. Next, suppose that $(1+\delta)v_L \geq$

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$c_L + \delta Q_L(\pi_0)$. By the monotonicity of $Q_L(\pi)$, we have $\hat{\pi}_1^R \geq \pi_0$. Again, since $\pi_1^{ND} \leq \pi_0$, Lemma B.4 implies that $\pi_1^R = \hat{\pi}_1^R$. $\qquad \square$

In Figure 2, for $c_L = c_L^{high}$, the intersection $\hat{\pi}_1^R$ is smaller than π_1^{ND}. Thus, according to Lemma B.4, the equilibrium rejection belief is $\pi_1^R = \pi_1^{ND}$. In Lemma B.4, we have determined the equilibrium rejection belief without explicitly characterizing the first-period bidding equilibrium. In particular, when $\hat{\pi}_1^R > \pi_0$, the buyers' bidding equilibrium may not be the same as the static equilibrium. Utilizing the characterization of the rejection belief, we next derive the first-period bidding equilibrium.

B.4.2 First-Period Bidding Equilibrium

Above construction assumes that the H-seller accepts any high offers with probability one. We need to ensure that this is indeed the case in equilibrium. As explained in the main text, when the rejection belief π_1^R is sufficiently high, the H-seller may prefer to reject a high offer that is close to c_H. To prevent such a deviation, the first-period bidding equilibrium must be adjusted from the static bidding equilibrium.

Definition B.1. *We say that the first-period bidding equilibrium is the ξ-static equilibrium if: (i) the buyers' bidding behavior and the H-seller's acceptance behavior are identical to those in the static equilibrium with c_H replaced by $\tilde{c}_H = c_H + \xi$; and (ii) the L-seller accepts any offer $p_1 \geq c_H$ with probability one and rejects the v_L-offer with some probability.*

The ξ-static equilibrium is the same as the static equilibrium with c_H replaced by $\tilde{c}_H$, except that the L-seller may reject the v_L-offer with some probability. The buyers' bidding behavior remains the same even when the L-seller rejects the v_L-offer, because they expect zero profit from bidding v_L. When ξ is zero, the H-seller accepts any high offer with probability one, so the first-period bidding equilibrium coincides exactly with the static equilibrium.

To build intuition, we begin with an informal argument for determining ξ. In the first period, the buyers with a high signal bid $c_H + \xi$ with $\xi \geq 0$. The constant ξ is pinned down by the following indifference condition:

$$c_H + \xi + \delta Q_H(\pi_1^D) = c_H + \delta Q_H(\pi_1^R),$$

which means that the H-seller is indifferent between accepting and rejecting the offer $c_H + \xi$. If there is $\xi \geq 0$ that solves the indifference condition with $c_H + \xi$ in the support of the h-buyers' bids, then the ξ-static bidding equilibrium is the equilibrium of the first-period bidding stage. Buyers do not have the incentive to deviate from the ξ-static bidding by submitting a bid in $[c_H, \tilde{c}_H)$, as the H-seller would reject such an offer—and importantly, the H-seller's strategy is sequentially rational.

The above argument delivers the key economic intuition for determining ξ, but it remains incomplete, because it ignores the fact that each ξ corresponds to a different first-period bidding equilibrium. Despite this complication, we show below that the intuition used to determine ξ remains valid.

We use the following notation. Let $\pi_1^D(\xi)$ denote the belief upon acceptance of a high offer when the first-period bidding equilibrium is the ξ-static equilibrium. The belief $\pi_1^d(\xi), d \in$

$\{D, ND\}$ is given by the disclosure/non-disclosure belief under the voluntary regime with c_H replaced by $\tilde{c}_H$ (see Table 3).

The following proposition shows that there exists a unique value of ξ that eliminates the H-seller's incentive to reject a high offer, while preserving buyers' incentives to follow the prescribed bidding strategy. The first part identifies conditions under which $\xi = 0$ (i.e., no adjustment to the static equilibrium). The second part explains how to determine ξ when such an adjustment is required.

Proposition B.1 (First-Period Bidding Equilibrium under Mandatory Disclosure). *Assume that* $\pi_0 \in (\underline{\pi}, \pi^\dagger)$.

(i) Suppose that $\hat{\pi}_1^R \leq \pi_1^D(0)$. *Then, the first-period bidding equilibrium is the* 0-*static equilibrium.*

(ii) Suppose that $\hat{\pi}_1^R > \pi_1^D(0)$. *The constant* $\xi > 0$ *is uniquely determined by*

$$c_H + \xi + \delta Q_H(\pi_1^D(\xi)) = c_H + \delta Q_H(\hat{\pi}_1^R), \tag{11}$$

if the solution exists in $\xi \in [0, \bar{\xi}]$, *where* $\bar{\xi} := V(\pi_1^h) - c_H$. *Otherwise, the buyers bid only* v_L *in the first period, and any* $\xi > \bar{\xi}$ *supports the same bidding equilibrium, in which Case 1 applies.*

The L-seller's acceptance probability of the v_L-offer, ϕ, is determined by

$$\hat{\pi}_1^R = \frac{\pi_1^{ND}(\xi)}{(\pi_1^{ND}(\xi) + (1 - \pi_1^{ND}(\xi))(1 - \phi))}.$$

In the second case, the L-seller's indifference condition holds with equality: $(1 + \delta)v_L = c_L + \delta Q_L(\hat{\pi}_1^R)$.

Proof of Proposition B.1. To prove the first claim, suppose that $\hat{\pi}_1^R \leq \pi_1^D(0)$. By Lemma B.5, we have $\pi_1^R \leq \pi_1^D(0)$. Thus the H-seller accepts any high offer $p_1 \geq c_H$ with probability one, and the 0-static equilibrium is the first-period bidding equilibrium.

Consider the case $\hat{\pi}_1^R > \pi_1^D(0)$.

Step 1. The L-seller's incentive. We show that the L-seller accepts any high offer $p_1 \geq c_H$ regardless of ξ. Consider an offer $p_1 \geq c_H$. Accepting and disclosing this offer gives the L-seller a payoff of $p_1 + \delta Q_L(\pi_1^D(\xi))$, while rejecting gives $c_L + \delta Q_L(\pi_1^R)$. If $\pi_1^D(\xi) \geq \pi_1^R$, then the result is immediate; thus consider the case of $\pi_1^D(\xi) < \pi_1^R$. In this case, the L-seller's indifference must hold with equality: $(1 + \delta)v_L = c_L + \delta Q_L(\pi_1^R)$. Moreover, by the monotonicity of Q_L, we have

$$p_1 + \delta Q_L(\pi_1^D(\xi)) \geq p_1 + \delta Q_L(0) > (1 + \delta)v_L.$$

Combining the preceding inequalities, we have that $p_1 + \delta Q_L(\pi_1^D(\xi)) > c_L + \delta Q_L(\pi_1^R)$, i.e., the L-seller strictly prefers to accept the high offer p_1.

Step 2. The H-seller's incentive. If there is no solution $\xi < V(\pi_1^h) - c_H = \bar{\xi}$ to (11), then H-seller rejects any high offers that the first-period buyers with h-signals are willing to

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bid.[43] In this case, any $\xi \geq \bar{\xi}$ supports the same equilibrium outcome, in which only v_L is offered in the first period.

We show that there is at most one solution to (11) in $\xi \in [0, V(\pi_1^h) - c_H]$. Observe that the left-hand side of equation (11) is strictly increasing in ξ. To see this, note that $\pi_1^D(\xi)$ is constant with respect to ξ when $c_H + \xi$ is Case 4 or 3 and is increasing if it is Case 2 (Lemma B.2). Therefore, the left-hand side of (11) is strictly increasing in ξ. When $\xi = 0$, the inequality (11) becomes

$$c_H + \delta Q_H(\pi_1^D(0)) < c_H + \delta Q_H(\hat{\pi}_1^R),$$

which is satisfied by the monotonicity of Q_H over $[\underline{\pi}, 1]$. Therefore, there is at most one solution over $\xi \in [0, V(\pi_1^h) - c_H]$.

In the ξ-static equilibrium, by construction, the H-seller accepts any high offer $p_1 \geq \tilde{c}_H = c_H + \xi$ and rejects any offers $p_1 < \tilde{c}_H$ with probability one. In particular, any offer $p_1 \in [c_H, \tilde{c}_H)$ does not arise in the equilibrium, so this constitutes the H-seller's equilibrium strategy.

Step 3. The buyers' incentive. The ℓ-bidders' incentive is the same under the original static bidding equilibrium. Moreover, the h-bidders do not have an incentive to deviate to bidding $x \in [c_H, \tilde{c}_H)$, because the H-seller rejects these offers in equilibrium. □

Remark B.1. *If $\pi_0 \leq \underline{\pi}$ (Case 1), then no high offer $p_1 \geq c_H$ occurs on-path, so the first-period bidding equilibrium corresponds to the 0-static equilibrium.*

Proposition B.1 suggests that sometimes the H-seller's incentive to reject a high offer is so strong that the first-period bidding equilibrium becomes as inefficient as it gets: the buyers offer v_L with probability one, and the H-seller never trades in the first period. The following corollary identifies the conditions under which this occurs.

Corollary B.1 (Extreme Inefficiency). *Suppose that the following is satisfied for some $\delta \in (0, 1]$:*

$$\delta > \frac{V(\pi_1^h) - c_H}{Q_H(\pi_1^R) - Q_H(\pi_1^h)}. \tag{C1}$$

Then, only v_L is offered in the first period (i.e., $\xi > \bar{\xi}$).

Proof. Suppose C1 holds. By Lemma B.5, we have $\pi_1^R = \hat{\pi}_1^R$. It suffices to show that the condition (11) is not satisfied for all $\xi \in [0, \bar{\xi}]$; in particular that

$$\bar{\xi} + \delta Q_H(\pi_1^D(\bar{\xi})) < \delta Q_H(\pi_1^R).$$

Since $\pi_1^D(\cdot) \leq \pi_1^h$ and $\bar{\xi} = V(\pi_1^h) - c_H$ into the left-hand side, the inequality is satisfied if

$$V(\pi_1^h) - c_H + \delta Q_H(\pi_1^h) < \delta Q_H(\pi_1^R),$$

which is equivalent to C1. □

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[43]When $\tilde{c}_H = c_H + \xi$ is in Case 2, the maximal on-path offer is $V(\pi_1^h)$. When it is in Case 3, the maximal on-path offer across all ξ is $\sup_{\xi < \bar{\xi}} [\mathbb{P}(\ell \mid h)(c_H + \xi) + \mathbb{P}(h \mid h)V(\pi_1^{hh})] = V(\pi_1^h)$.

For condition C1 to hold, $V(\pi_1^h) - c_H < Q_H(\pi_1^R) - Q_H(\pi_1^h)$ is necessary. This inequality is more likely to hold when c_L is low. In this case, the L-seller has a weak incentive to reject the v_L-offer, so π_1^R is high. The H-seller thus has a strong incentive to reject high offers. As a result, the first-period bidding equilibrium becomes inefficient, because the buyers do not submit a bid that would be rejected by the H-seller.

B.4.3 Summary

We are now ready to prove Proposition 2. Most of the work has been done in the preceding analysis.

Proof of Proposition 2. Given the relevant first-period ξ-static bidding equilibrium, buyers' strategies are optimal by construction, and we only need to check the seller's acceptance incentives.

(i) $(1 + \delta)v_L < c_L + \delta Q_L(\pi_0)$.

   The L-seller has no profitable deviation by construction. For the H-seller, rejecting $p_1 = v_L$ yields a payoff of $c_H + \delta Q_H(\pi_1^R)$, whereas accepting it yields $v_L + \delta c_H$, which is strictly lower than $c_H + \delta Q_H(\pi_1^R)$. If $p_1 \geq c_H$, then the H-seller has no profitable deviation by Proposition B.1. By Lemma B.5, the belief after rejection of the v_L-offer is lower than the prior: $\pi_1^R < \pi_0$.

(ii) $(1 + \delta)v_L \in [c_L + \delta Q_L(\pi_0), c_L + \delta Q_L(\pi_1^D(0)))$.

   The seller's deviation incentives are the same as in the previous case. By Lemma B.5, the belief after rejection of the v_L-offer is higher than the prior: $\pi_1^R \geq \pi_0$.

(iii) $(1 + \delta)v_L \in [c_L + \delta Q_L(\pi_1^D(0)), c_L + \delta v_H)$.

   The L-seller accepts the v_L-offer with some probability by construction. In Proposition B.1, we have verified the L- and H-seller's incentives to accept high offers $p_1 \geq \tilde{c}_H$. Moreover, the H-seller rejects $p_1 = v_L$ with probability one, as acceptance yields the posterior belief 0. By Proposition B.1, $\pi_1^R \geq \pi_1^D(0)$.

   $\square$

B.5 Proof of Theorem 1 and Proposition A.1

Here we prove our main welfare result. We combine the statement of Theorem 1, which compares the mandatory and the voluntary regime, and that of Proposition A.1, which analyzes the opaque regime.

Theorem B.2. *Welfare between the three regimes has the following properties:*

(i) (Voluntary Dominates) *Suppose that $\pi_0 \in (\overline{\pi}, \pi^\dagger)$ (high-intermediate initial belief, Case 3). Then, welfare is strictly higher under the voluntary regime than under the mandatory and the opaque regime if $\pi_0 > \pi^*$ for some $\pi^* \in (\overline{\pi}, \pi^\dagger)$ and either:*

   (i.a) *c_L is high enough: $c_L > c_L^*$ for $c_L^* \in (0, v_L)$ determined in equilibrium; or*

   (i.b) *the condition C1 holds—in particular, c_L is low enough.*

(ii) (Mandatory Dominates) *Suppose that $\pi_0 < \bar{\pi}$ (low-intermediate initial belief, Cases 2 and 1). Then, welfare is strictly higher under the mandatory regime than the voluntary regime, if*

    (ii.a) $\pi_0 \in (\underline{\pi}, \bar{\pi})$ *(Case 2) and* $(1+\delta)v_L \leq c_L + \delta Q_L(\pi_1^D)$.

    (ii.b) $\pi_0 \leq \underline{\pi}$ *(Case 1).*

    Moreover, the mandatory regime dominates the other regimes if $\pi_0 \leq \underline{\pi}$.

(iii) (Opaque Dominates) *Suppose that $\pi_0 \geq \pi^{\dagger}$ (high initial belief, Case 4). Then, welfare is equal in the mandatory and voluntary regimes. The opaque regime dominates the other two regimes.*

We prove Part (i) and Part (ii) in Sections B.5.2 and B.5.3, respectively. In Section B.5.4, we prove Part (iii). Although the economic intuition explained in the main text illustrates the core ideas—the seller's signaling incentives distort trade—the technical details are intricate.

In the proof, we manipulate c_L, which controls the seller's signaling incentives in the mandatory regime. Under the mandatory regime, the L-seller has a stronger incentive to mimic the H-seller by rejecting low offers when c_L is higher. Conversely, when c_L is lower, the H-seller has a stronger incentive to mimic the L-seller by rejecting high offers. Thus the probability of trade under the mandatory regime depends directly on c_L. Under the voluntary regime, however, c_L does not affect equilibrium at all, since signaling through strategic offer rejection is muted. Similarly, the opaque regime's welfare is independent of c_L.

Therefore, by understanding how the mandatory regime's equilibrium changes with c_L, we can compare welfare across regimes. But this alone is insufficient. We must also understand the *rate* at which the trade probability changes with c_L. This is the major challenge in the proof. To see why, note that welfare *directly* changes when c_L changes. For example, as c_L increases, the L-seller rejects low offers more aggressively, reducing the trade probability. Yet, as c_L approaches v_L, the potential gain from trade, $v_L - c_L$, diminishes. If $v_L - c_L$ declines quickly enough relative to the decrease in trade probability, then welfare under the mandatory regime may not decline as c_L rises.

In the proof below, most of the technical analysis is devoted to addressing this aspect.

B.5.1 Welfare and Expected Payoffs

For a given seller type θ and realized trade outcome, welfare equals v_θ if trade occurs and c_θ if it does not. Equivalently, welfare is the sum of the seller's and buyers' payoffs. We note that the trade price does not affect welfare.

Our welfare analysis focuses on the expected discounted sum of welfare at the start of the game. Let $w_\theta(\pi_{t-1}) := \mathbb{E}[z_t v_\theta + (1 - z_t)c_\theta \mid \theta; \pi_{t-1}]$ denote the expected welfare in state θ in period t, given the initial period-t belief (i.e., prior to the bidding stage) π_{t-1}. The expected discounted sum of welfare in state θ under regime $\mathcal{R}$ is given by $W_\theta^{\mathcal{R}}(\pi_0) := w_\theta(\pi_0) + \delta\mathbb{E}[w_\theta(\pi_1) \mid \pi_0]$, where $\mathcal{R} \in \{\mathcal{M}, \mathcal{D}, \mathcal{O}\}$ represents the mandatory disclosure, voluntary disclosure, and opaque regimes, respectively.

Sometimes it is more convenient to calculate welfare in terms of the expected payoffs. Let $u_{B,s_i}(\pi_{t-1})$ be the buyer i's expected payoff in period t, given the initial period-t belief π_{t-1}. Let $u_B(\pi_{t-1})$ be a buyer's expected payoff before the signal realization. Then, $U_B^{\mathcal{R}}(\pi_0) :=$

$2(u_B(\pi_0) + \delta\mathbb{E}[u_B(\pi_1) \mid \pi_0])$ is the buyers' expected payoffs. Let $u_{S,\theta}(\pi_{t-1}) := \mathbb{E}[z_t p_t + (1 - z_t)c_\theta \mid \theta; \pi_{t-1}]$ be the θ-seller's expected equilibrium payoff in period t, given the initial belief π_{t-1}. $U_{S,\theta}^{\mathcal{R}}(\pi_0) := u_{S,\theta}(\pi_0) + \delta\mathbb{E}[u_{S,\theta}(\pi_1) \mid \pi_0]$ is the θ-seller's expected discounted sum of payoffs in regime $\mathcal{R}$. We have the following relationship between welfare and expected payoffs:

$$\pi_0 W_H^{\mathcal{R}}(\pi_0) + (1 - \pi_0)W_L^{\mathcal{R}}(\pi_0) = \pi_0 U_{S,H}^{\mathcal{R}}(\pi_0) + (1 - \pi_0)U_{S,L}^{\mathcal{R}}(\pi_0) + U_B^{\mathcal{R}}(\pi_0).$$

We further note that, under the voluntary and opaque regimes, the L-seller always trades in both periods; therefore, c_L does not affect welfare under these regimes. In the proofs below, we use this fact and focus on how the signaling incentives and thus welfare under the mandatory regime change with c_L.

B.5.2 Proof of Part (i)

Throughout this Section B.5.2, we assume that $\pi_0 \in (\overline{\pi}, \pi^\dagger)$ (Case 3).

(i.a) The L-Seller has Strong Mimicry Incentives: c_L is high We prove Part (i.a) by showing that the L-seller under the mandatory regime rejects the v_L-offer with probability one in the first period when c_L is sufficiently high. When this occurs, the L-seller trades more frequently under the voluntary regime. By contrast, the H-seller trades with the same probability in both regimes, yielding higher welfare under the voluntary regime.

To that end, we first show that, when π_0 is in the upper region of Case 3 of Lemma 1, the disclosure belief is Case 4 whereas the non-disclosure belief is Case 2.

Lemma B.6. *There exists $\pi^* \in (\overline{\pi}, \pi^\dagger]$ such that, for all $\pi_0 \in (\pi^*, \pi^\dagger)$, disclosure belief is Case 4 and non-disclosure belief is Case 2; that is,*

$$\pi_1^D > \pi^\dagger \text{ and } \pi_1^{ND} \in (\underline{\pi}, \overline{\pi})$$

Proof. For any $\pi_0 \in (\overline{\pi}, \pi^\dagger)$, the disclosure belief is given by (see Table 3)

$$\pi_1^D = \frac{2q(1-q)\pi_0 + \{\pi_0 q^2 + (1-\pi_0)(1-q)^2\}\pi_1^{hh}}{2q(1-q) + \{\pi_0 q^2 + (1-\pi_0)(1-q)^2\}} > \pi_0.$$

In particular, this belief updating is continuous at $\pi_0 = \pi^\dagger$. Since $\pi_1^D > \pi^\dagger$ when $\pi_0 = \pi^\dagger$, there exists π_0' such that, for all $\pi_0 \in (\pi_0', \pi^\dagger)$, $\pi_1^D > \pi^\dagger$. Similarly, at $\pi_0 = \pi^\dagger$, the non-disclosure belief is $\pi_1^{ND} = \pi_1^{\ell\ell} = \overline{\pi}$. Thus, there exists π_0'' such that, for all $\pi_0 \in (\pi_0'', \pi^\dagger)$, $\pi_1^{ND} \in (\underline{\pi}, \overline{\pi})$. Take $\pi^* = \max\{\pi_0', \pi_0''\}$ to complete the proof. □

Now we show that, when c_L is high enough, the L-seller strictly prefers to reject low offers in the first period. Intuitively, when c_L is high enough, the L-seller's mimicry cost is low, so she rejects aggressively; the rejection belief then becomes low, and there is no need to adjust the bidding equilibrium.

Lemma B.7. *Consider the mandatory disclosure regime. Suppose that $\pi_0 \in (\pi^*, \pi^\dagger)$, where π^* is defined in Lemma B.6. There exists $c_L^* < v_L$ such that, for all $c_L \in (c_L^*, v_L)$, the L-seller*

strictly prefers to reject low offers in the first period ($\phi = 0$). In this case, the first-period bidding equilibrium is the 0-static equilibrium ($\xi = 0$).

Proof. By Lemma B.6, we have $Q_L(\pi_1^{ND}) > v_L$ (see Table 2). Therefore, there exists $c_L^* < v_L$ such that, for all $c_L \in (c_L^*, v_L)$, we have $(1 + \delta)v_L < c_L + \delta Q_L(\pi_1^{ND})$. This implies that $\phi = 0$ (see Lemma B.4). By Proposition B.1, the first-period bidding equilibrium is the 0-static equilibrium ($\xi = 0$). □

With these preparations, we prove that the voluntary regime dominates the mandatory and the opaque regime.

Proof of Theorem B.2, Part (i.a). Suppose that $\pi_0 \in (\pi^*, \pi^\dagger)$ and $c_L \in (c_L^*, v_L)$. We can compute welfare in each regime as follows for $\theta = L$:

$$W_L^{\mathcal{O}}(\pi_0) = (1 + \delta)v_L,$$
$$W_L^{\mathcal{D}}(\pi_0) = (1 + \delta)v_L,$$
$$W_L^{\mathcal{M}}(\pi_0) = \mathbb{P}_\xi(p_1 = v_L \mid L)(\phi v_L + (1 - \phi)c_L + \delta v_L) + \mathbb{P}_\xi(p_1 \geq c_H \mid L)(1 + \delta)v_L,$$

where $\mathbb{P}_\xi$ denotes the probability distribution over the first-period offers in the mandatory regime under the ξ-static bidding equilibrium. Under the opaque and voluntary regimes, the L-seller always trades. Under the mandatory regime, welfare depends on whether the first-period offer p_1 is low ($p_1 = v_L$) or high ($p_1 \geq c_H$). By Lemma B.7, $\phi = 0$ and $\xi = 0$, so

$$W_L^{\mathcal{D}}(\pi_0) > W_L^{\mathcal{M}}(\pi_0). \tag{12}$$

For the H-seller, we have the following:

$$W_H^{\mathcal{O}}(\pi_0) = (1 + \delta)w_H(\pi_0),$$
$$W_H^{\mathcal{D}}(\pi_0) = w_H(\pi_0) + \delta \left[\mathbb{P}(p_1 = v_L \mid H)w_H(\pi_1^{ND}) + \mathbb{P}(p_1 \geq c_H \mid H)w_H(\pi_1^{D}) \right],$$
$$W_H^{\mathcal{M}}(\pi_0) = w_H(\pi_0) + \delta \left[\mathbb{P}_\xi(p_1 = v_L \mid H)w_H(\pi_1^{R}) + \mathbb{P}_\xi(p_1 \geq c_H \mid H)w_H(\pi_1^{D}) \right].$$

By Lemma B.7, we have $\pi_1^{R} = \pi_1^{ND}$ and $\xi = 0$, implying that

$$W_H^{\mathcal{D}}(\pi_0) = W_H^{\mathcal{M}}(\pi_0). \tag{13}$$

Combining (12) and (13), we conclude that the voluntary regime dominates the mandatory regime.

To see that the voluntary regime dominates the opaque regime, using $w_H(\pi_0) = \mathbb{P}(p_1 = v_L \mid H)c_H + \mathbb{P}(p_1 \geq c_H \mid H)v_H$, we have

$$W_H^{\mathcal{D}} - W_H^{\mathcal{O}} = \delta \left[\mathbb{P}(p_1 = v_L \mid H)(\underbrace{w_H(\pi_1^{ND})}_{>c_H} - c_H) + \mathbb{P}(p_1 \geq c_H \mid H)(\underbrace{w_H(\pi_1^{D})}_{=v_H} - v_H) \right].$$

Therefore, $W_H^{\mathcal{D}} - W_H^{\mathcal{O}} > 0$. Since $W_L^{\mathcal{D}} = W_L^{\mathcal{O}}$, the voluntary regime dominates the opaque regime. □

(i.b) The L-Seller has Weak Mimicry Incentives: c_L is Low   The above proof of the welfare comparison between the voluntary and opaque regime only relies on $\pi_0 > \pi^*$; thus the voluntary regime dominates the opaque regime regardless of c_L. Thus, we only need to prove that the voluntary regime dominates the mandatory regime.

Proof of Theorem B.2, Part (i.b). Let $\pi_0 > \pi^*$ and suppose that the condition C1 holds. Under the mandatory regime, the first-period offer is v_L for sure by C1. Therefore,

$$W_L^{\mathcal{M}}(\pi_0) = (\phi v_L + (1 - \phi)c_L) + \delta v_L.$$

Since $\phi < 1$ and $W_L^{\mathcal{D}}(\pi_0) = (1 + \delta)v_L$, we have $W_L^{\mathcal{D}}(\pi_0) > W_L^{\mathcal{M}}(\pi_0)$.
    For the H-seller, we have the following:

$$W_H^{\mathcal{D}}(\pi_0) = w_H(\pi_0) + \delta\left[\mathbb{P}(p_1 = v_L \mid H)w_H(\pi_1^{ND}) + \mathbb{P}(p_1 \geq c_H \mid H)w_H(\pi_1^{D})\right],$$
$$W_H^{\mathcal{M}}(\pi_0) = c_H + \delta w_H(\pi_1^{R}).$$

The difference is

$$\begin{aligned}
W_H^{\mathcal{D}} - W_H^{\mathcal{M}} &= \mathbb{P}(p_1 = v_L \mid H)(c_H + \delta w_H(\pi_1^{ND})) + \mathbb{P}(p_1 \geq c_H \mid H)(v_H + \delta w_H(\pi_1^{D})) \\
&\quad - (c_H + \delta w_H(\pi_1^{R})) \\
&= \delta\mathbb{P}(p_1 = v_L \mid H)(\underbrace{w_H(\pi_1^{ND})}_{>c_H} - \underbrace{w_H(\pi_1^{R})}_{\leq v_H}) \\
&\quad + \mathbb{P}(p_1 \geq c_H \mid H)\left(v_H - c_H + \delta(\underbrace{w_H(\pi_1^{D}) - w_H(\pi_1^{R})}_{\geq 0})\right) \\
&> (\mathbb{P}(p_1 \geq c_H \mid H) - \delta\mathbb{P}(p_1 = v_L \mid H))(v_H - c_H) \\
&> 0.
\end{aligned}$$

We used the fact that $\pi_1^{D} > \pi^{\dagger}$, so $v_H = w_H(\pi_1^{D}) \geq w_H(\pi_1^{R})$. □

B.5.3   Proof of Part (ii)

(ii.a) Low-intermediate initial belief   Consider $\pi_0 \in (\underline{\pi}, \overline{\pi})$ (Case 2). Suppose that $(1 + \delta)v_L \leq c_L + \delta Q_L(\pi_1^{D})$. By Lemma B.4 and Proposition B.1, the first-period bidding equilibrium under the mandatory regime is the 0-static equilibrium, and the L-seller's indifference condition holds with equality. Moreover, the non-disclosure belief π_1^{ND} is always such that Case 1 of Lemma 1 applies.

Lemma B.8. *Suppose $\pi_0 \in (\underline{\pi}, \overline{\pi})$. Then, $\pi_1^{ND} < \underline{\pi}$.*

Proof. Using the expression (9), it is straightforward to show that π_1^{ND} is decreasing in π_0 on this region. Since $\lim_{\pi_0 \downarrow \underline{\pi}} \pi_1^{ND} = \underline{\pi}$ and $\pi_1^{ND} \leq \pi_0$, the claim follows. □

Proof of Part (ii.a)–Low-Intermediate Belief. It is more convenient to calculate the expected payoffs than to compute welfare directly. The L-seller's expected payoffs are

$$U^{\mathcal{D}}_{S,L}(\pi_0) = U^{\mathcal{M}}_{S,L}(\pi_0) = \mathbb{P}(p_1 = v_L \mid L)(1+\delta)v_L + \mathbb{P}(p_1 \geq c_H \mid L)(\mathbb{E}[p_1 \mid p_1 \geq c_H, L] + \delta u_{S,L}(\pi^D_1)).$$

To see why, consider first the voluntary regime. If the first-period offer is low ($p_1 = v_L$), then the L-seller accepts and withholds the offer. In the second period, since $\pi^{ND}_1 < \underline{\pi}$ (Lemma B.8), the buyers offer only v_L. This outcome corresponds to the first term of the above expression. If instead the first-period offer is high ($p_1 \geq c_H$), then the L-seller accepts and discloses the offer. In the second period, the posterior is π^D_1, which gives an expected payoff of $u_{S,L}(\pi^D_1)$. This path corresponds to the second term. Now consider the mandatory regime. If the first-period offer is v_L, then the L-seller is indifferent between accepting and rejecting, so the payoff from a low offer is $(1 + \delta)v_L$ by the equilibrium condition. If instead the first-period offer is high, the evolution of beliefs is identical to the voluntary regime. Therefore, the expected payoff is also identical.

For the H-seller, the expected payoffs are

$$U^{\mathcal{D}}_{S,H}(\pi_0) = \mathbb{P}(p_1 = v_L \mid H)(1 + \delta)c_H + \mathbb{P}(p_1 \geq c_H \mid H)(\mathbb{E}[p_1 \mid p_1 \geq c_H, H] + \delta u_{S,H}(\pi^D_1)),$$
$$U^{\mathcal{M}}_{S,H}(\pi_0) = \mathbb{P}(p_1 = v_L \mid H)(c_H + \delta u_{S,H}(\pi^R_1)) + \mathbb{P}(p_1 \geq c_H \mid H)(\mathbb{E}[p_1 \mid p_1 \geq c_H, H] + \delta u_{S,H}(\pi^D_1)).$$

The expression for the voluntary regime directly follows from the earlier discussion of the L-seller's payoff. Under the mandatory regime, if the first-period offer is v_L, the H-seller rejects this for sure, inducing belief π^R_1. Since $\pi^R_1 > \underline{\pi}$, we have $u_{S,H}(\pi^R_1) > c_H$. If the first-period offer is high, then the evolution of beliefs mirrors that in the voluntary regime. Therefore, $U^{\mathcal{M}}_{S,H} > U^{\mathcal{D}}_{S,H}$.

Finally, we compute the buyers' payoffs. In the first period, buyers' expected payoffs are zero under both regimes (see Table 2). If the first-period offer is low, then second-period buyers receive an expected payoff of zero under the voluntary regime. Under the mandatory regime, however, the low offer may be rejected with positive probability, allowing second-period buyers to earn a possibly positive expected payoff. If the first-period offer is high, then posterior beliefs in the second period are the same under both regimes, and so are the buyers' expected payoffs.

In conclusion, we have that

$$U^{\mathcal{D}}_{S,L}(\pi_0) = U^{\mathcal{M}}_{S,L}(\pi_0), \; U^{\mathcal{M}}_{S,H}(\pi_0) > U^{\mathcal{D}}_{S,H}(\pi_0), \text{and } U^{\mathcal{M}}_B \geq U^{\mathcal{D}}_B.$$

Hence, the mandatory regime dominates the voluntary regime. □

(ii.b) Low initial belief Suppose that $\pi_0 \leq \underline{\pi}$. Again, we compute expected payoffs instead of welfare directly. Since the first-period offer is v_L for sure, there is no belief updating under the voluntary regime (see Table 3). Thus the voluntary and opaque regimes have the same welfare. In particular, the H-seller is always excluded from trade:

$$U^{\mathcal{D}}_{S,L}(\pi_0) = U^{\mathcal{O}}_{S,L}(\pi_0) = (1 + \delta)v_L$$
$$U^{\mathcal{D}}_{S,H}(\pi_0) = U^{\mathcal{O}}_{S,H}(\pi_0) = (1 + \delta)c_H.$$

Buyers in both periods earn zero expected profits under the voluntary and opaque regimes.

Now consider the mandatory regime. Since $\pi_1^{ND} < \underline{\pi}$, the L-seller's indifference condition always holds with equality. Thus the L-seller's expected payoff is $U_{S,L}^{\mathcal{M}}(\pi_0) = (1+\delta)v_L$. For the H-seller, as in part (ii.a), by $\pi_1^R > \underline{\pi}$, we have $U_{S,H}^{\mathcal{M}}(\pi_0) = c_H + \delta u_{S,H}(\pi_1^R) > (1+\delta)c_H$. The buyers' expected payoffs are weakly positive. Hence, the mandatory regime dominates both the voluntary and opaque regime. This completes the proof of Part (ii).

B.5.4   Proof of Part (iii)

Suppose that $\pi_0 \geq \pi^\dagger$. First, we argue that voluntary and mandatory regimes yield the same welfare. The first-period welfare is maximized under both regimes, because any on-path offers are high and are accepted. In the second period, under both regimes, if the first-period offer is $p_1 > V(\pi_1^{\ell\ell})$, then the second-period belief is π_1^D; if the first-period offer is $p_1 = V(\pi_1^{\ell\ell})$, then the second-period belief is $\pi_1^{ND} = \pi_1^{\ell\ell}$. Since the second-period welfare is determined by the belief π_1, we conclude that both regimes yield the same welfare in the second period as well.

Under the opaque regime, first-period welfare replicates in the second period. As a result, trade is always efficient. Therefore, the opaque regime weakly dominates—and strictly so if $\pi_1^{ND} < \pi^\dagger$—both the mandatory and voluntary regimes.

Proof of Corollary 1

If $\pi_0 \geq \pi^\dagger$, then the mandatory and the voluntary regimes are outcome-equivalent, and therefore the trading frequencies are the same. Suppose that $\pi_0 \in (\pi^*, \pi^\dagger)$ and $c_L \in (c_L^*, v_L)$, where π^* and c_L^* are from Theorem B.2 (Lemmas B.6 and B.7). This corresponds to case (i.a) of Theorem B.2, so the result immediately follows. In particular, given $\theta = L$, the seller rejects first-period low offers with probability one under the mandatory regime, while she accepts them for sure under the voluntary regime. Given $\theta = H$, the trade probabilities are the same under both regimes. In total, trade occurs more frequently under the voluntary regime.

Proof of Proposition 3

We prove the proposition by backward induction for a finite T and then show that the proposed strategy constitutes an equilibrium in the limit $T \to \infty$.

Finite $T < \infty$.

Fix $T < \infty$. In period T and $T-1$, the proposed strategy profile forms an equilibrium by Proposition 1. Consider period $t = T-2$. Suppose that the L-seller receives a high offer $p_{T-2} \geq c_H$. Accepting and disclosing this yields

$$p_{T-2} + \delta \left[u_{S,L}(\pi_{T-2}^D) + \delta \mathbb{E}[u_{S,L}(\pi_{T-1}) \mid L; \pi_{T-2}^D] \right],$$

while deviating to concealing the trade yields

$$p_{T-2} + \delta \left[u_{S,L}(\pi_{T-2}^{ND}) + \delta \mathbb{E}[u_{S,L}(\pi_{T-1}) \mid L; \pi_{T-2}^{ND}] \right].$$

Since $\pi_{T-1}^{D} > \pi_{T-1}^{ND}$, the deviation is not profitable.[44] Similarly, if the L-seller receives a low offer $p_{T-2} = v_L$, accepting and concealing the trade dominates. The H-seller's incentives can be verified similarly. Iterating this argument back to period 1, we conclude that accepting and disclosing a high offer is optimal for the L-seller in all periods.

Limit $T \to \infty$.

In each period t, as $T \to \infty$, the continuation payoff converges uniformly to

$$Q_\theta(\pi_{t-1}) := \sum_{\tau=t}^{\infty} \delta^{\tau-t} \mathbb{E}[u_{S,\theta}(\pi_{\tau-1}) \mid \pi_{t-1}].$$

Since Q_θ is increasing, the same argument as in the two-period case shows that the proposed strategy profile constitutes an equilibrium in the limit $T \to \infty$.

Proof of Proposition 4

We employ a similar approach to that used in the proof of Proposition 3. The mandatory regime is more complex, because we must analyze both the L-seller's acceptance probability of low offers, ϕ_t, and the bidding-equilibrium parameter ξ_t in each period. Because the analysis proceeds by backward induction, it is convenient to index strategies and continuation payoffs by the number of periods remaining, which we denote by the superscript "(t)". For example, $\phi^{(t)} := \phi_{T-t}$ denotes the L-seller's acceptance probability in period $T-t$.

Finite $T < \infty$.

First, we show that the L-seller never accepts a low offer with probability one on the equilibrium path.

Lemma B.9. *For any $T \geq 2$, we have*

$$\frac{1-\delta^T}{1-\delta} v_L \leq c_L + \delta \frac{1-\delta^{T-1}}{1-\delta} v_H. \tag{14}$$

Proof. Since $v_L < v_H$, by adding terms to the inequality from Assumption 1, we have

$$(1+\delta)v_L + \sum_{t=2}^{T-1} \delta^t v_L < c_L + \delta v_H + \sum_{t=2}^{T-1} \delta^t v_H.$$

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[44]Since $u_{S,L}$ is weakly increasing in the belief, we have $u_{S,L}(\pi_{T-2}^{D}) \geq u_{S,L}(\pi_{T-2}^{ND})$. For the continuation payoff, note that $\pi_{T-1}^{D}|_{d_{T-2}=D} > \pi_{T-1}^{D}|_{d_{T-2}=ND}$. Moreover, $\pi_{T-1}^{ND}|_{d_{T-2}=D} > \pi_{T-1}^{ND}|_{d_{T-2}=ND}$ if $\pi_{T-1}^{ND}|_{d_{T-2}=ND} \geq \underline{\pi}$. If $\pi_{T-1}^{ND}|_{d_{T-2}=ND} < \underline{\pi}$ then $u_{S,L}(\pi_{T-1}^{ND}|_{d_{T-2}=ND}) = v_L$. Hence, we have $\mathbb{E}[u_{S,L}(\pi_{T-1}) \mid L; \pi_{T-2}^{D}] \geq \mathbb{E}[u_{S,L}(\pi_{T-1}) \mid L; \pi_{T-2}^{ND}]$.

This expression is equivalent to (14). □

Since (14) holds for all T, the L-seller never accepts a low offer with probability one in any period.

Next, we show a long-horizon analog of Lemma B.3. At the points where Q_L is discontinuous, we pick the buyers' mixing strategy to guarantee the existence of the solution.

Lemma B.10. *For any T, there is a solution $\hat{\pi}_1^R$ to the equation*

$$\sum_{t=0}^{T-1} \delta^t v_L = c_L + \delta Q_L^{(T-1)}(\hat{\pi}_1^R). \tag{15}$$

Proof. Using $\sum_{t=0}^{T-1} \delta^t = \frac{1-\delta^T}{1-\delta}$, rewrite the equation as

$$v_L = \frac{1-\delta}{1-\delta^T} c_L + \frac{\delta}{1-\delta^T} \underbrace{(1-\delta)Q_L^{(T-1)}(\hat{\pi}_1^R)}_{\in (1-\delta^{T-1})[v_L, v_H]}.$$

Note that $Q_L^{(T-1)} \in \sum_{t=0}^{T-2} \delta^t [v_L, v_H]$. For $\hat{\pi}_1^R \to 0$, the RHS is a convex combination of c_L and $(1-\delta^{T-1})v_L < v_L$, so the LHS is larger. For $\hat{\pi}_1^R \to 1$, we have $Q_L^{(T-1)} \to \sum_{t=0}^{T-2} \delta^t v_H$, so Lemma B.9 implies that the RHS is larger than the LHS. By appropriately choosing the buyers' mixing strategy as in Lemma B.3, $Q_L^{(T-1)}$ is effectively continuous, so there exists a solution to (15). □

We now explicitly construct the equilibrium by backward induction.

Proof of Proposition 4, Finite $T < \infty$. In period $T-1$, using the two-period result (Proposition 2), we have the equilibrium strategies as described in the proposition. In particular, we obtain $\phi^{(1)}(\pi_{T-2})$ and $\xi^{(1)}(\pi_{T-2})$ for all beliefs $\pi_{T-2} \in (0,1)$. Consider period $t = T - 2$. From Lemma B.10, let $\hat{\pi}_{T-2}^R$ be a solution to

$$(1 + \delta + \delta^2)v_L = c_L + \delta Q_L^{(2)}(\hat{\pi}_{T-2}^R).$$

The continuation payoff is written as

$$Q_L^{(2)}(\hat{\pi}_{T-2}^R) = u_{S,L}(\hat{\pi}_{T-2}^R; \xi^{(1)}(\hat{\pi}_{T-2}^R)) + \delta \mathbb{E}[Q_L^{(1)}(\pi_{T-1}) \mid L; \hat{\pi}_{T-2}^R],$$

where $u_{S,L}(\hat{\pi}_{T-2}^R; \xi^{(1)})$ is the L-seller's expected payoff in the $\xi^{(1)}$-static bidding equilibrium. By the same argument as in the two-period case, $\pi_{T-2}^R = \max\{\pi_{T-2}^{ND}, \hat{\pi}_{T-2}^R\}$. From π_{T-2}^R, we obtain $\phi^{(2)}(\pi_{T-2}) \in [0,1)$ for all beliefs $\pi_{T-2} \in (0,1)$. The seller and buyers do not have deviation incentives by construction.

Iterating this argument back to period 1, we can construct $\phi^{(t)}(\pi_{t-1})$ and $\xi^{(t)}(\pi_{t-1})$ for all $t < T$ and beliefs $\pi_{t-1} \in (0,1)$, which constitutes an equilibrium. □

Limit $T \to \infty$.

Given the above construction of equilibrium for finite T, consider the limit $T \to \infty$.[45] The continuation payoff converges uniformly: $Q_\theta^{(T)} \to Q_\theta$ as $T \to \infty$. Since $\xi^{(T)}$ and $\phi^{(T)}$ are both bounded,[46] we can extract a convergent subsequence for each belief π. By construction, the limits $\xi(\pi)$ and $\phi(\pi)$ constitute a stationary Markov equilibrium in the limit $T \to \infty$. In particular, the equilibrium conditions in the limit are given by

$$\begin{cases} \frac{1}{1-\delta} v_L = c_L + \delta Q_L(\pi^R), \\ \xi + \delta Q_H(\pi^D) = \delta Q_H(\pi^R), \end{cases}$$

where π^R is the rejection belief and π^D is the belief after disclosure of a high offer. For each initial belief π, the limits $\xi(\pi)$ and $\phi(\pi)$ satisfy the above conditions.

Proof of Proposition A.2

As discussed in the main text, once the belief reaches Case 1 (i.e., $\pi_{t-1} < \underline{\pi}$), buyers offer only v_L in period t, the H-seller rejects the offer, and the L-seller accepts it and withholds the trade. Since the buyer offers are not based on private signals and no disclosure ensues regardless of the seller's type, the belief is not updated in period t: $\pi_t = \pi_{t-1}$. Consequently, a *DOWN* cascade occurs only without learning: the belief cannot reach 0.

Below, we first derive the minimal number of periods needed for a *DOWN* cascade to occur, T_k. Then we prove the results for the $T \to \infty$ case.

Positive probability of a *DOWN* cascade

Fix $\theta \in \{L, H\}$. With positive probability, any finite number of consecutive signal realizations (ℓ, ℓ) occur.

When π_0 is Case 2, π_1^{ND} is Case 1 (Lemma B.8). When π_0 is Case 3, it is straightforward to verify that at most two successive (ℓ, ℓ) signal realizations lead to Case 2. Therefore, $T_2^{\mathcal{P}} = 2$ and $T_3^{\mathcal{P}} = 3$ are the minimum number of periods needed for a positive probability of a *DOWN* cascade for each case.

Suppose that π_0 is Case 4. In this case, when π_0 is high enough, π_1^{ND} can still stay in Case 4. Consider the belief π_t after t successive (ℓ, ℓ) signal realizations (i.e., non-disclosures). Once the period-t^* initial belief π_{t^*-1} reaches Case 3, it takes at most two more periods for the belief to reach Case 1 by $T_3^{\mathcal{P}} = 3$. Therefore, we have $T_4^{\mathcal{P}} = t^* + 2$.

To derive t^*, we write the likelihood ratio of π_t as

$$\frac{\pi_t}{1-\pi_t} = \frac{\pi_0}{1-\pi_0} \left(\frac{1-q}{q} \right)^{2t}, \quad t \geq 0$$

The belief π_t is monotonically decreasing in t; it reaches Case 3 in the first period t in which

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[45]The argument mirrors that in Fudenberg and Tirole (1991, Chap. 13), because our game is "continuous at infinity" and the buyers are short-lived.

[46]Note that $\xi^{(T)} \in [0, v_H]$.

$\pi_{t-1} < \pi^\dagger$ (or equivalently $\pi_t = \pi_t^{\ell\ell} < \overline{\pi}$), so

$$\frac{\pi_{t^*}}{1-\pi_{t^*}} < \frac{\overline{\pi}}{1-\overline{\pi}} \iff \left(\frac{q}{1-q}\right)^{2t^*} > \frac{\pi_0/(1-\pi_0)}{\overline{\pi}/(1-\overline{\pi})} \iff t^* > \frac{1}{2}\frac{\log\frac{\pi_0/(1-\pi_0)}{\overline{\pi}/(1-\overline{\pi})}}{\log\frac{q}{1-q}}.$$

Hence,

$$T_4^{\mathcal{D}} = \left\lceil \frac{1}{2}\frac{\log\frac{\pi_0/(1-\pi_0)}{\overline{\pi}/(1-\overline{\pi})}}{\log\frac{q}{1-q}} \right\rceil + 2.$$

Belief convergence when $T \to \infty$

Now consider the case $T \to \infty$. Regardless of θ and π_0, the belief reaches Case 4 with positive probability after a sequence of high offers. Therefore, without loss, suppose that π_0 is Case 4.

First, consider $\theta = H$. We show that, with positive probability, the belief converges to 1 while remaining in Case 4:

$$\mathbb{P}(\pi_t \xrightarrow{t\to\infty} 1 \text{ and } \pi_t \geq \pi^\dagger, \forall t) > 0. \tag{16}$$

Consider an auxiliary Markov process $\{\hat{\pi}_t\}_{t\geq 0}$ defined by $\hat{\pi}_0 = \pi_0$, where the transition probabilities are taken as if the process always stays in Case 4. Then, conditional on the event that the original process indeed stays in Case 4, the original and the auxiliary process have the same distribution: $\mathbb{P}(\{\pi_t\} = \{\hat{\pi}_t\} \mid \pi_t \geq \pi^\dagger, \ \forall t) = 1$. Therefore, it suffices to show (16) for $\{\hat{\pi}_t\}$.

The log-likelihood ratio process of $\{\pi_t\}$ can be expressed as

$$\log\frac{\hat{\pi}_{t+1}}{1-\hat{\pi}_{t+1}} = \begin{cases} \log\frac{\hat{\pi}_t}{1-\hat{\pi}_t} + \log\frac{\mathbb{P}((s_i,s_j)\neq(\ell,\ell)|H)}{\mathbb{P}((s_i,s_j)\neq(\ell,\ell)|L)} & \text{w.p. } \mathbb{P}((s_i,s_j) \neq (\ell,\ell) \mid H), \\ \log\frac{\hat{\pi}_t}{1-\hat{\pi}_t} + \log\frac{\mathbb{P}((s_i,s_j)=(\ell,\ell)|H)}{\mathbb{P}((s_i,s_j)=(\ell,\ell)|L)} & \text{w.p. } \mathbb{P}((s_i,s_j) = (\ell,\ell) \mid H). \end{cases}$$

The increments are i.i.d with positive expectation, so the log-likelihood ratio process is a random walk with positive drift. Thus it is immediate that $\log((\hat{\pi}_t)/(1-\hat{\pi}_t)) \to \infty$, so $\hat{\pi}_t \to 1$. Furthermore, the following result implies that (16) holds.

Fact. *Let X_t be a random walk with positive drift with a fixed starting point $X_0 = x_0$. Take any $b < x_0$. Then, $\mathbb{P}(X_t > b, \forall t) > 0$.*

Next, consider $\theta = L$. By an analogous argument, we can show that the log-likelihood ratio process is a random walk with negative drift within each case. Therefore, the belief eventually reaches Case 1 with probability one.

Proof of Proposition A.3

Consider the equilibrium constructed in Proposition 4. Unlike the voluntary regime, there is no "absorbing" beliefs in which the belief is no longer updated.

Lemma B.11. *Under the mandatory regime, the belief is always updated on the equilibrium path: $\pi_t \neq \pi_{t-1}$ as long as $\pi_{t-1} \in (0,1)$.*

Proof. Consider period t with the initial belief $\pi_{t-1} \in (0,1)$. If the period-t bidding equilibrium is Cases 2–4, then with positive probability a high offer p_{high} arrives. The seller accepts this offer, and the public belief is updated to $\pi_t^D > \pi_{t-1}$.

When the bidding equilibrium is Case 1, buyers offer only v_L. It suffices to show that the L-seller does not reject the v_L-offer with probability one. Suppose, toward a contradiction, that the L-seller rejects v_L with probability one. Then, it must be that $\xi^{(T-t)}(\pi_{t-1}) = 0$ in the period-t bidding equilibrium, because the bidding adjustment is necessary only when the rejection belief is higher than the initial belief, which occur only when the L-seller accepts the v_L-offer with a positive probability. Hence, $\pi_{t-1} < \underline{\pi}$.

From the equilibrium condition for the L-seller, we have

$$c_L + \delta Q_L^{(T-t)}(\pi_{t-1}) \geq \sum_{\tau=0}^{T-t} \delta^\tau v_L, \tag{17}$$

where the continuation belief remains at $\pi_{t-1} < \underline{\pi}$ after rejection. Thus the bidding equilibrium is again Case 1 in the next period. In period $t+1$, if the L-seller is indifferent between accepting and rejecting v_L, then the indifference condition implies that $Q_L^{(T-t)}(\pi_{t-1}) = \sum_{\tau=0}^{T-t-1} \delta^\tau v_L$, which contradicts (17). If instead the L-seller strictly prefers to reject v_L, then we must have $c_L + \delta Q_L^{(T-t-1)}(\pi_t) \geq \sum_{\tau=0}^{T-t-1} \delta^\tau v_L$ with $\pi_t = \pi_{t-1}$. We can repeat the previous argument until (i) the L-seller is indifferent between accepting and rejecting v_L, leading to a contradiction as above, or (ii) we reach period T, in which case we have $Q_L^{(T-t)}(\pi_{t-1}) = c_L$ in the original condition (17), again leading to a contradiction. $\square$

Lemma B.11 immediately implies that cascade without learning cannot occur under the mandatory regime. Therefore, it suffices to consider only cascades with learning below.

Cascade in finite time

Fix $T < \infty$. The H-seller never accepts a v_L-offer, so her type is never revealed in finite time and a cascade never occurs.

Suppose $\theta = L$. If π_0 is Case 1, 2, or 3, then there is a positive probability that $p_1 = v_L$. Under Case1, the L-seller accepts the low offer with positive probability (see Lemma B.11), leading to a *DOWN* cascade with learning. Hence, $T_1^{\mathcal{M}} = 2$.

Under Case 2, she also accepts the low offer with positive probability. To see this, suppose toward a contradiction that the L-seller rejects the low offer with probability one. Then, the continuation belief is $\pi_1^{ND} < \underline{\pi}$, in which case the continuation payoff is $\sum_{\tau=0}^{T-2} \delta^\tau v_L$ by Lemma B.11. However, this violates the equilibrium condition in the first period. Hence, $T_2^{\mathcal{M}} = 2$.

Under Case 3, the L-seller may reject the low offer with probability one in the first period. In that case, the resulting rejection belief, $\pi_1^R = \pi_1^{ND}$, falls into Case 2. Hence, $T_3^{\mathcal{M}} = 3$.

If π_0 is Case 4, then the belief evolves in the same way as the voluntary regime until it reaches Case 3. After this event, the previous paragraph implies that it takes at most two periods for a *DOWN* cascade to occur. Hence, $T_4^{\mathcal{M}} = T_4^{\mathcal{D}}$.

Belief convergence when $T \to \infty$

It suffices to show that the belief converges to 1 with probability one conditional on $\theta = H$. When π_t is Case 4, the same argument as in the voluntary regime shows that the belief converges to 1 with positive probability while remaining in Case 4. The next argument shows that this actually occurs with probability one.

When π_0 is Cases 1—3, there is a positive probability that the buyer offers v_L in the first period (regardless of ξ). Let π^R denote the belief after a rejection in the infinite-horizon game. From Lemma B.10, the rejection belief is not Case 1. Therefore, in the second period, there is a positive probability of a high offer p_{high}, which the H-seller accepts, and the belief is updated upward. By iterating this argument, we conclude that the belief reaches Case 4 with probability one in finite time.

Hence, an UP cascade occurs with probability one under the mandatory regime when $\theta = H$.

Proof of Theorem 2

Let $\pi_0 \in (\pi^*, \pi^\dagger)$, where π^* is from Theorem B.2 (Lemma B.6). Suppose that for c_L sufficiently high, the L-seller rejects the v_L offer in the first period with probability one. Then the second-period belief is the same under both regimes, since $\pi_1^R = \pi_1^{ND}$. Hence, given $\theta = L$, the welfare gain of the voluntary regime relative to the mandatory regime is[47]

$$W_L^{\mathcal{D}} - W_L^{\mathcal{M}} = \mathbb{P}(p_1 = v_L \mid L)(v_L - c_L) + \underbrace{0}_{\text{second period}} + o(\delta),$$

where $\lim_{\delta \to 0} o(\delta)/\delta = 0$, since any welfare effect beyond the second period is of order δ^2. When $\theta = H$, the welfare gain of the voluntary regime is zero in both periods. Therefore, the total welfare gain of the voluntary regime relative to the mandatory regime is

$$W^{\mathcal{D}} - W^{\mathcal{M}} = (1 - \pi_0)\mathbb{P}(p_1 = v_L \mid L)(v_L - c_L) + o(\delta). \tag{18}$$

Thus, for δ small enough, the welfare gain from the voluntary regime in the first period dominates, and the expression is strictly positive.

This argument, however, is incomplete. We assumed that the L-seller rejects the low offer with probability one in the first period when c_L is sufficiently high. But as δ becomes smaller, the L-seller's incentive to reject the low offer weakens, requiring a higher c_L to sustain rejection. Yet as c_L rises, the term $v_L - c_L$ in (18) declines, potentially offsetting the welfare gain.

To complete the argument, we must show that the required increase in c_L is slower than the rate at which δ shrinks. To establish this, we show that the L-seller has a stronger incentive to reject low offers as the number of remaining periods increases.

Lemma B.12. *Suppose that for some $c_L \in (0, v_L)$, the L-seller rejects the low offer with probability one when only one period remains. Then, for the same c_L, the L-seller also rejects*

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[47]Since the L-seller rejects the low offer with probability one, the first-period bidding equilibrium is the 0-static equilibrium.

the low offer with probability one when $T - 1$ periods remain for any $T > 2$.

Proof. By Lemma B.7, there exists $c_L^*(\delta) \in (0, v_L)$ such that, for any $c_L > c_L^*(\delta)$, the L-seller rejects the low offer with probability one if only one period remains (i.e., the first period in $T = 2$ case):

$$c_L^*(\delta) := (1 + \delta)v_L - \delta Q_L^{(1)}(\pi_1^{ND}). \tag{19}$$

Here, we explicitly write the dependence of c_L^* on δ. Since the L-seller's continuation value in each period is at least v_L, when $T - 1$ periods remain with $T > 2$, the continuation payoff satisfies

$$Q_L^{(T-1)}(\pi_1^{ND}) > Q_L^{(1)}(\pi_1^{ND}) + (\delta + \delta^2 + \cdots + \delta^{T-2})v_L. \tag{20}$$

Therefore,

$$\delta \left(Q_L^{(T-1)}(\pi_1^{ND}) - \sum_{t=0}^{T-2} \delta^t v_L \right) > \delta \left(Q_L^{(1)}(\pi_1^{ND}) - v_L \right) = v_L - c_L^*(\delta),$$

where the first inequality follows by subtracting v_L from both sides of (20) and rearranging, and the second equality follows from (19). Hence,

$$\sum_{t=0}^{T-1} \delta^t v_L < c_L + \delta Q_L^{(T-1)}(\pi_1^{ND}),$$

which implies that the L-seller rejects the low offer with probability one in the first period of the T-period game. $\qquad\square$

Proof of Theorem 2. From the preceding discussion, at $c_L = c_L^*(\delta)$, the welfare gain is

$$W^{\mathcal{D}} - W^{\mathcal{M}} = (1 - \pi_0)\mathbb{P}(p_1 = v_L \mid L)(v_L - c_L^*(\delta)) + o(\delta).$$

Since the gap $v_L - c_L^*(\delta)$ shrinks at order of δ, the welfare gain in (18) remains positive for sufficiently small δ. More precisely,

$$(1 - \pi_0)\mathbb{P}(p_1 = v_L \mid L) \underbrace{\delta \left(Q_L^{(1)}(\pi_1^{ND}) - v_L \right)}_{=v_L - c_L^*(\delta)} + o(\delta)$$

$$= \delta \left[(1 - \pi_0)\mathbb{P}(p_1 = v_L \mid L)(Q_L^{(1)}(\pi_1^{ND}) - v_L) + o(1) \right],$$

which is strictly positive for $\delta < \bar{\delta}$ and $c_L > c_L^*(\bar{\delta})$ for some $\bar{\delta} > 0$. $\qquad\square$

C Refinement and Uniqueness of Equilibrium

In Section 3.2, we mention that the voluntary disclosure equilibrium in Proposition 1 is the unique PBE that survives a particular refinement. Similarly, Proposition 2 states that the equilibrium in the mandatory disclosure regime uniquely survives a standard refinement.

In this appendix, we formally introduce the refinement concepts and show that our PBE outcomes in the main text are essentially unique. For each regime, we use what we believe is the most natural refinement. Under the voluntary regime, signaling (mimicry by disclosure) is costless, in which case the standard refinement is the GPFE criterion (Bertomeu and Cianciaruso, 2018; Glode et al., 2018). Under the mandatory regime, signaling (mimicry by rejection) is costly, in which case the D1 criterion is a corresponding refinement (Banks and Sobel, 1987).

C.1 Voluntary Disclosure Regime

C.1.1 Tie-breaking

For expositional convenience, we assume the following tie-breaking rule:

Assumption 3. *When the seller is indifferent between disclosing and not disclosing, she chooses not to disclose, and this is common knowledge among the seller and buyers.*

This tie-breaking assumption is innocuous, and it is solely to simplify the exposition. When the seller is indifferent between disclosing and not disclosing, the posterior belief upon disclosure is the same regardless of disclosure. By assuming that the seller does not disclose in such a case, we can avoid some notational clutter. This assumption can be justified by assuming that there is a small disclosure cost, and the equilibrium is the limit when the disclosure cost tends to zero.

C.1.2 Existence of Multiple PBE

In Proposition 1, we conjectured the seller's strategy and showed that it is indeed an equilibria. In this section, we identify other possible equilibria.

Since the θ-seller accepts any first-period offer $p_1 \geq c_\theta$ (Lemma B.1), buyers' bidding behavior is described by Lemma 1. The equilibrium offers can be classified as either low or high offer, denoted by p_{low} and p_{high}, respectively. Specifically, when Cases 1–3 of Lemma 1 apply (i.e., $\pi < \pi^\dagger$), let $p_{\text{low}} = v_L$ and $p_{\text{high}} \geq c_H$. When Case 4 applies (i.e., $\pi \geq \pi^\dagger$), let $p_{\text{low}} = V(\pi^{\ell\ell})$ and $p_{\text{high}} > V(\pi^{\ell\ell})$, where $V(\pi) := \pi v_H + (1-\pi)v_L$.

Lemma C.1. *In any PBE of the voluntary disclosure game, the seller's disclosure outcome does not reveal her type. That is, $\pi_1^d \notin \{0, 1\}$.*

Proof. By Lemmas 1 and B.1, the L-seller accepts all on-path offers. Consequently, the L-seller's disclosure strategy affects only her continuation payoff. Therefore, if disclosure of trade at p_1 or non-disclosure fully reveals the seller's type, then the L-seller strictly benefits from mimicking the H-seller's disclosure strategy. $\square$

Accordingly, there are three possible PBE outcomes:[48]

- (Weak separation) Both types accept and disclose p_{high}. Both types withhold p_{low}.
- (Pooling by non-disclosure) Both types withhold all offers.

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[48]Another possible disclosure outcome is that the seller discloses only the low offer. However, it is easy to verify that this cannot be a PBE, because the H-seller strictly benefits by deviating to non-disclosure.

- (Pooling by disclosure) Both types accept and disclose p_{high} and p_{low}.

In Proposition 1, we have shown that the weak separation is a PBE. In addition, the "pooling by non-disclosure" equilibrium always exists. To see why, we describe the seller's action at the disclosure stage by a message $m \in \{p_{\text{low}}, p_{\text{high}}, ND\}$, where $m = p$ denotes disclosure of offer p and $m = ND$ denotes non-disclosure. Let $\pi^o(m) \in [0,1]$ be an off-path belief for an off-path message m. If both types of the seller withhold all offers, the second-period belief is π_0 for sure. Therefore, any off-path belief $\pi^o(p) \leq \pi_0$ for $p \in \{p_{\text{low}}, p_{\text{high}}\}$ deters a deviation.

The "pooling by disclosure" equilibrium is inefficient in the sense that the H-seller is forced to accept low offers when the initial belief is Cases 1–3. When the initial belief is Case 4, the equilibrium outcome under pooling by disclosure is the same as that under weak separation; thus we consider the case in which the initial belief is Cases 1–3. In the pooling-by-disclosure equilibrium, the H-seller makes a loss by accepting the low offer. As such, this equilibrium can be supported only when the deviation results in sufficiently low belief. In particular, the H-seller does not deviate to rejection if

$$v_L + \delta Q_H(\pi_0) \geq c_H + \delta Q_H(\pi^o(ND)). \tag{21}$$

for some $\pi^o(ND) \leq \pi_0$. Depending on the parameter values, there may not be an off-path belief that satisfies (21), but if it does, "pooling by disclosure" is a PBE.

We now introduce a standard equilibrium refinement and show that the weak-separation outcome is the unique PBE outcome.

C.1.3   Grossman-Perry-Farrell Equilibrium (GPFE)

We use the Grossman-Perry-Farrell equilibrium (GPFE) refinement, developed by Grossman and Perry (1986) and Farrell (1993). Roughly speaking, an equilibrium fails the GPFE criterion if there is a set of types who wish to deviate, given that the off-path belief is formed by Bayes's rule under the correct conjecture of deviators. This refinement is more appropriate in disclosure games, where common refinements such as the intuitive criterion and the D1 criterion fail to eliminate unnatural equilibria. Indeed, Bertomeu and Cianciaruso (2018) show that the GPFE criterion selects a unique equilibrium for generic disclosure games, while the intuitive criterion does not.[49]

To formally state the solution concept, note that there are four "types" of the seller from the perspective of second-period buyers, depending on the quality and the offer type. Let $\Omega := \{L, H\} \times \{p_{\text{low}}, p_{\text{high}}\}$ be the effective type space. Each quality-offer type (θ, p_1) determines whether to accept the offer and whether to disclose it. Note that a message $m = p_1$ can be sent by a type who has accepted the offer, while a message $m = ND$ can be sent by any type.

To define GPFE, we follow Bertomeu and Cianciaruso (2018) and Sobel (2020) to define

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[49]Our disclosure subgame is not necessarily covered in Bertomeu et al. (2022) because the seller's payoff depends on the price as well.

a *self-signaling set* C by[50]

$$C(m) = \begin{cases} \{(\theta, p_1) \in \Omega \mid U^*_{S,\theta}(p_1) < p_1 + \delta Q_\theta(\pi_C(p_1))\} & \text{if } m = p_1 \in \{p_{\text{low}}, p_{\text{high}}\}, \\ \{(\theta, p_1) \in \Omega \mid U^*_{S,\theta}(p_1) < p_1 + \delta Q_\theta(\pi_C(ND))\} & \text{if } m = ND. \end{cases}$$

where $U^*_{S,\theta}(p_1)$ is the (θ, p_1)-seller's equilibrium payoff and Q_θ is the θ-seller's continuation payoff given an off-path belief $\pi_C(m)$. Importantly, the off-path belief $\pi_C(m)$ is determined by the Bayes' law: it is the posterior belief over $\{L, H\}$ conditional on C and m. Therefore, a self-signaling set C exists if the second-period buyers who observe an off-path message m believe that the message is from quality-offer types in C and only those types wish to deviate to m. We call a PBE that does not admit a self-signaling set C a GPFE.

C.1.4 Applying the GPFE Criterion

Pooling by non-disclosure   We argue that $C(p_{\text{high}}) = \{(L, p_{\text{high}}), (H, p_{\text{high}})\}$ constitutes a self-signaling set. For this set, the posterior is $\pi_C(p_{\text{high}}) = \pi_1^D$, while the on-path belief is $\pi_0 < \pi_1^D$. Thus, both types in $C(p_{\text{high}})$ wish to deviate to $m = p_{\text{high}}$. Moreover, the seller with $p_1 = p_{\text{low}}$ cannot deviate to this message. Hence, pooling by non-disclosure cannot be a GPFE.

Pooling by disclosure   The only off-path message is $m = ND$. We argue that

$$C(ND) = \{(L, p_{\text{low}}), (H, p_{\text{low}}), (L, p_{\text{high}}), (H, p_{\text{high}})\}$$

constitutes a self-signaling set. For this set, the posterior is $\pi_C(ND) = \pi_0$. The on-path posterior belief upon disclosure is also π_0. The L-seller strictly prefers to deviate by Assumption 3, and the H-seller strictly prefers to deviate because she is making a loss by accepting the low offer. Hence, this equilibrium, if it exists, is not a GPFE.

Weak Separation   We show that there is no self-signaling set. The only off-path message is $m = p_{\text{low}}$. By a similar argument as Lemma C.1, a self-signaling set cannot reveal type. Thus it suffices to consider $C(p_{\text{low}}) = \{(L, p_{\text{low}}), (H, p_{\text{low}})\}$, which induces $\pi_C(p_{\text{low}}) = \pi_1^{ND}$. Since the seller in $C(p_{\text{low}})$ receives the same posterior belief on the equilibrium path but acceptance is potentially costly, the seller does not deviate to this off-path message. The seller with $p_1 = p_{\text{high}}$ cannot deviate to this message. Hence, there is no self-signaling set, and this equilibrium uniquely survives the GPFE criterion.

C.2 Mandatory Disclosure Regime

C.2.1 Pooling Equilibrium

Under the mandatory regime, in any PBE, any on-path offer $p_1 \geq c_H$ is accepted by the H-seller. This is because a first-period buyer with the high signal does not have an incentive

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[50]From Lemma C.1, we only need to consider the deviation to "accept and withhold" in case of $m = ND$.

to submit a bid that is rejected for sure by the H-seller. Therefore, when the initial belief is Case 4 (i.e., $\pi \geq \pi^\dagger$), the unique PBE outcome is that all offers are accepted.

When the initial belief is in Cases 1–3 (i.e., $\pi < \pi^\dagger$), there are two types of PBE outcomes:[51]

- The H-seller rejects $p_1 = v_L$.
- The H-seller accepts all on-path offers.

In the main text, we have shown that the first type of equilibrium always exists. To see when the second type of equilibrium exists, suppose that the H-seller accepts all on-path offers. On the equilibrium path, the H-seller who received $p_1 = v_L$ makes a loss, so this equilibrium can be supported if the off-path belief is sufficiently low. In particular, this can be an equilibrium if, for some off-path belief upon rejection, $\pi^o \in [0, 1]$,

$$v_L + \delta Q_H(\pi_1^{ND}) \geq c_H + \delta Q_H(\pi^o), \tag{22}$$

where π_1^{ND} is the belief upon acceptance of $p_1 = v_L$ (Table 3).[52]

If the inequality (22) does not hold for any $\pi^o \in [0, 1]$, the PBE outcome in Proposition 2 is unique. For example, if the initial belief is low enough so that $\pi_1^{ND} \leq \underline{\pi}$, then we have $Q_H(\pi_1^{ND}) = c_H$. In this case, the inequality (22) cannot hold, because $Q_H \geq c_H$. When the pooling equilibrium exists, we show that it does not survive the D1 criterion.

C.2.2 D1 Criterion

To define the D1 criterion following Cho and Kreps (1987), for a given PBE, let $D_\omega(m) \subset [0, 1]$ ($D_\omega^0(m) \subset [0, 1]$) be the set of off-path beliefs that type $\omega \in \Omega$ weakly (strictly) prefers to deviate to the off-path message m. Denote by $\Omega(m) \subset \Omega$ the set of types who can send the message m. If there are types $\omega \neq \omega'$ in $\Omega(m)$ such that

$$D_\omega(m) \cup D_\omega^0(m) \subsetneq D_{\omega'}(m),$$

then the D1 criterion requires the off-path belief assign probability one to type ω'. Intuitively, type ω' is "more likely" to deviate than ω.

We show that the PBE outcome in which the H-seller accepts all on-path offers, if it exists, does not survive the D1 criterion. To see this, suppose that the initial belief is Cases 1–3 and that the H-seller accepts all on-path offers. Let π^o be the off-path belief upon rejection that supports this equilibrium. The equilibrium conditions of the L- and H-seller are

$$v_L + \delta Q_H(\pi_1^{ND}) \geq c_H + \delta Q_H(\pi^o),$$
$$v_L + \delta Q_L(\pi_1^{ND}) \geq c_L + \delta Q_L(\pi^o).$$

Since $c_H > v_L$, the H-seller strictly wishes to deviate for any $\pi^o \geq \pi_1^{ND}$. In contrast, since $c_L < v_L$, the L-seller wishes to deviate only when π^o is sufficiently higher than π_1^{ND}.

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[51]A strategy in which the H-seller randomizes over accepting and rejecting $p_1 = v_L$ cannot be a PBE, because the H-seller's indifference implies that the L-seller strictly prefers to accept $p_1 = v_L$.

[52]The inequality (22) implies that $\pi_1^{ND} > \pi^o$, so the L-seller strictly prefers to accept $p_1 = v_L$.

Therefore, this equilibrium fails the D1 criterion.