SUPPLEMENTARY MATERIAL for

The Gatekeeping Expert's Dilemma

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C Additional Details

I provide additional details omitted in the main text.

C.1 Optimal Silence Set

In this section, I give a general characterization of the auditor-optimal silence set under precise communication. Recall that an auditor-optimal silence set is derived by maximizing (9) subject to (10), or equivalently,

$$\begin{aligned} \min_{a,\,b}\ & \mathbb{P}(X \in [a,b])\mathbb{E}[\min\{(r_0 - X)^2, (\pi^R)^2\}] + \mathbb{P}(X \notin [a,b])(\pi^R)^2 \\ & \text{s.t.}\ |X - r_0| \geq \pi^R, \end{aligned}$$

where $r_0 = r([a,b])$ is the manager's report upon the auditor's silence.

The main goal of this section is to show that an optimal silence set pools all the low-end values of X.

Proposition C.1. *Consider the precise communication case (i.e., $\mathcal{M} = \mathcal{M}^{\dagger}$). An auditor-optimal equilibrium is given by a silence set of the form $\mathcal{X}_0^* = [\underline{X}, b^*]$ for some $b^* \in (\underline{X}, \overline{X}]$.*

The result resembles the threshold disclosure equilibrium in the canonical disclosure models (Verrecchia, 1983; Dye, 1985). However, the underlying economic forces are distinct. In those models, the sender pools bad news because she wants to conceal it. In contrast, here the auditor pools low values of X ("bad news" for the manager) because doing so helps to weaken the manager's incentive to experiment. More specifically, without any communication (i.e., $\mathcal{X}_0 = [\underline{X}, \overline{X}]$), the manager's report $r_0 = r(\mathcal{X}_0)$ lies to the right of the modal value of X (assuming strict log-concavity). The auditor's payoff is strictly higher than $-(\pi^R)^2$ when $X \in [r_0 - \pi^R, r_0 + \pi^R]$. The auditor's expected payoffs improve if she can shift the interval $[r_0 - \pi^R, r_0 + \pi^R]$ to the left, because it increases the probability of this event. The auditor can achieve this by eliminating the high-end values of X from the silence set, as this curtails the manager's incentive to inflate the report.

I first show that an optimal silence set is an interval.

Lemma C.1. *Consider the precise communication case (i.e., $\mathcal{M} = \mathcal{M}^{\dagger}$). In any equilibrium, the silence set $\mathcal{X}_0$ is a connected subset (interval) of $[\underline{X}, \overline{X}]$.*

Proof. Let $r_0 := r(\mathcal{X}_0)$ be the manager's report when the auditor stays silent. Suppose toward a contradiction that $\mathcal{X}_0$ is not connected. Without loss of generality, let $\mathcal{X}_0 = [a_1, b_1] \cup [a_2, b_2]$ for some $a_1 < b_1 < a_2 < b_2$. The manager's reporting problem is

$$\max_{r \geq 0} \ \mathbb{P}(X \in [a_1, b_1] \mid \mathcal{X}_0) \, r A(r; [a_1, b_1]) + \mathbb{P}(X \in [a_2, b_2] \mid \mathcal{X}_0) \, r A(r; [a_2, b_2]), \tag{C1}$$

where $A(r; [a_i, b_i])$ is the acceptance probability when the message is $[a_i, b_i]$ for $i = 1, 2$.

Therefore, $[r_0 - \pi^R, r_0 + \pi^R]$ must lie entirely within either $[a_1, b_1]$ or $[a_2, b_2]$. Without loss, suppose $[r_0 - \pi^R, r_0 + \pi^R] \subset [a_1, b_1]$. Then for any $r \in [r_0 - \pi^R, r_0 + \pi^R]$, we have $A(r; [a_2, b_2]) = 0$. Thus, the solution to (C1) is equivalent to solving $\max_{r \geq 0} r A(r; [a_1, b_1])$. It follows that the auditor's expected loss is the same whether the silence set is $\mathcal{X}_0' = [a_1, b_1]$ or $\mathcal{X}_0$.[1] □

With this lemma, I now prove Proposition C.1.

Proof of Proposition C.1. From Lemma C.1, we know that an optimal silence set is an interval $[a, b]$. Thus it suffices to show that $a = \underline{X}$. Let r_0 be the report induced under an auditor-optimal equilibrium. The report is either $r_0 > a + \pi^R$ or $r_0 = a + \pi^R$ from Lemma 1. Consider the first case. The manager's response remains unchanged for any $\mathcal{X}_0' := [a', b]$ with $a' \leq a$. Therefore, setting $a = \underline{X}$ weakly improves the auditor's payoff. Suppose instead that $r_0 = a + \pi^R$. Since $b - a > 2\pi^R$ in equilibrium, the report r_0 must satisfy $r_0 = a + \pi^R > r_1^+$, where r_1^+ is from the proof of Lemma 1 (see (17)). Note that r_1^+ is the manager's report without any communication (i.e., $[a, b] = [\underline{X}, \overline{X}]$). From the proof of Lemma 1, we know that r_1^+ is in the region where the density f is decreasing. Hence, the auditor's expected loss under a strategy that induces $r_0 = a + \pi^R$ is strictly higher than that under a complete silence strategy. □

Given Proposition C.1, the auditor's expected loss can be written as a function of b only:

$$L^\dagger(b) := \mathbb{P}(X \leq b)\mathbb{E}[\min\{(r_0 - X)^2, (\pi^R)^2\}] + \mathbb{P}(X > b)(\pi^R)^2.$$

Minimizing $L^\dagger(b)$ subject to the equilibrium constraint $|X - r_0| \geq \pi^R$ gives the optimal right endpoint b^* of the silence set. As in the case of the uniform distribution, the solution need not be unique. However, when f is strictly log-concave, the solution is unique.[2]

Proposition C.2. *Suppose that f is strictly log-concave. There is a unique solution to*

$$\min_{b \geq \max\{\underline{b}, \underline{X} + 2\pi^R\}} L^\dagger(b),$$

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[1] Note that the manager's expected payoff is higher under the new silence set $\mathcal{X}_0'$.

[2] This does not imply that the optimal silence set is unique, as $a = \underline{X}$ is not the only choice of the left endpoint.

where $\underline{b}$ is the unique solution to

$$b - \pi^R = \frac{F(b) - F(b - 2\pi^R)}{f(b - 2\pi^R)}. \tag{C2}$$

The uniqueness of $\underline{b}$ solving (C2) is guaranteed by the log-concavity of f. The feasible set $\{b \mid b \geq \max\{\underline{b}, \underline{X} + 2\pi^R\}\}$ is derived from the equilibrium constraint $|X - r_0| \geq \pi^R$:

Lemma C.2. *Suppose that f is strictly log-concave. Then, $[r_0 - \pi^R, r_0 + \pi^R] \subset \mathcal{X}_0 = [\underline{X}, b] \iff b \geq \max\{\underline{b}, \underline{X} + 2\pi^R\}$.*

Proof. If $b < \underline{X} + 2\pi^R$, then the short-interval case of Lemma 1 applies. In this case $r_0 + \pi^R > b$, so $r_0 + \pi^R \notin \mathcal{X}_0$. Otherwise, the long-interval case applies, and the report satisfies $r_0 \leq b - \pi^R$ if and only if $r_2^+ \leq b - \pi^R$, where r_2^+ is defined in (17). This condition is equivalent to $b \geq \underline{b}$, because r_2^+ is the unique fixed point of $r = (F(b) - F(r - \pi^R))/f(r - \pi^R)$. □

Now I prove Proposition C.2 by showing that the loss function $L^\dagger(b)$ is quasi-convex over the feasible set.

Proof of Proposition C.2. Since $r_0 + \pi^R \leq b$ for $b \geq \max\{\underline{b}, \underline{X} + 2\pi^R\}$, the loss function can be written as

$$L^\dagger(b) = \int_{r_0 - \pi^R}^{r_0 + \pi^R} (r_0 - x)^2 f(x)dx + (F(r_0 - \pi^R) + 1 - F(r_0 + \pi^R))(\pi^R)^2$$

for $b \geq \max\{\underline{b}, \underline{X} + 2\pi^R\}$. The (left) derivative is given by

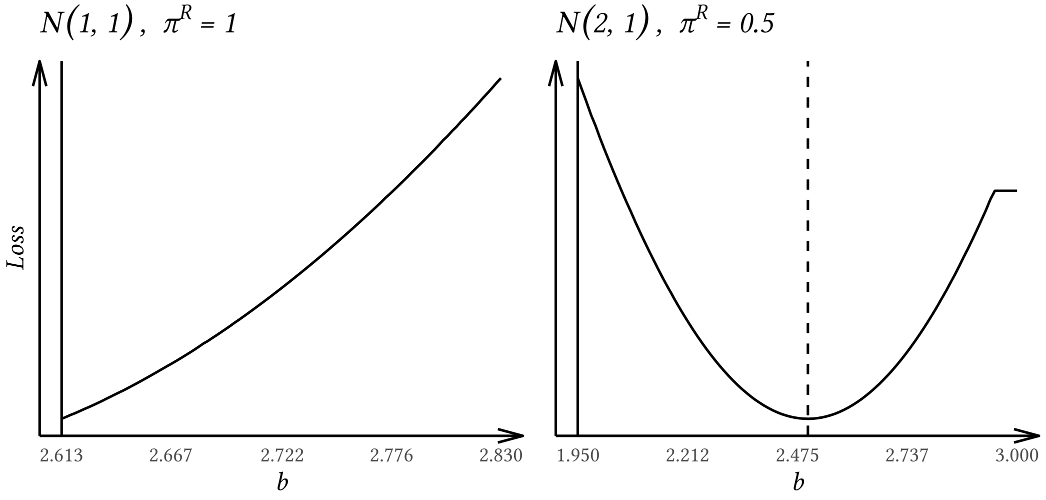
$$(L^\dagger)'(b) = 2\frac{\partial r_0}{\partial b} \int_{r_0 - \pi^R}^{r_0 + \pi^R} (r_0 - x) f(x)dx.$$

From strict log-concavity, there is a unique r that solves $r = \mathbb{E}[X \mid X \in [r - \pi^R, r + \pi^R]]$. Thus $(L^\dagger)'(b)$ changes sign only once from negative to positive as b increases. Hence, there is a unique minimizer $b^* \geq \max\{\underline{b}, \underline{X} + 2\pi^R\}$ of $L^\dagger(b)$. □

Proposition C.2 implies that, in general, strictly log-concavity ensures that the auditor's payoff in an auditor-optimal equilibrium is strictly higher than the no-communication payoff. Although analytical expressions for the loss function are not available in general, the above result guarantees that we can find the optimal b^* numerically.

Example 1 (Normal Distribution)**.** Suppose that X follows a normal distribution. Since $\underline{X} = -\infty$, the equilibrium constraint is simply $b \geq \underline{b}$. To derive the optimal silence set, one can first numerically solve (C2) to obtain $\underline{b}$, and then minimize $L^{\dagger}(b)$ over the range $b \geq \underline{b}$. Figure C.1 shows the loss functions $L^{\dagger}(b)$ for two sets of parameters. The solid vertical line indicates $\underline{b}$. The left panel corresponds to Example 2 in the main text. The auditor's expected loss is minimized at $b = \underline{b}$. The right panel illustrates the case in which the loss is minimized at an interior point in the feasible set. ∎

Figure C.1: Optimal Silence Set under Normal Distribution



Note: This figure plots the loss function $L^{\dagger}(b)$ when X follows a normal distribution. In the left panel, $X \sim \mathcal{N}(1,1)$ with $\pi^R = 1$. In the right panel, $X \sim \mathcal{N}(2,1)$ with $\pi^R = 0.5$.

C.2 Optimal Uniform-Uniform Partition

I provide a fully explicit solution to the optimal uniform partition problem (12) in the case of a uniform distribution. Let N^* and Δ^* be the solution, as defined in (13). I assume that $\pi^R \geq \underline{X}$, which is a sufficient condition for the uniform partition to be optimal.

Proposition C.3. *Let $X \sim \mathcal{U}[\underline{X}, \overline{X}]$ and $\pi^R \geq \underline{X}$. Define*

$$y := \frac{3\pi^R + 2(\overline{X} - \underline{X}) + \sqrt{9(\pi^R)^2 + 4(\overline{X} - \underline{X})^2}}{6\pi^R}. \tag{C3}$$

Then, a uniform partition is optimal. The optimal number of intervals is given by $N^* \in \{y-1, y\}$ if $y \in \mathbb{N}$ and $N^* = \lfloor y \rfloor$ otherwise.

Proof. We first ignore the constraint $(\overline{X} - \underline{X})/N \leq 2\pi^R$ in (13) and solve

$$\underset{N \in \mathbb{N}}{\arg\min} \left| \frac{\overline{X} - \underline{X}}{N} - 1.5\pi^R \right|. \tag{C4}$$

For each $k \geq 2$, define $\overline{\pi}(k)$ as the value of π^R at which k- and $(k-1)$-partitions yield the same payoff for the auditor:

$$\frac{\overline{X} - \underline{X}}{k} - 1.5\overline{\pi}(k) = 1.5\overline{\pi}(k) - \frac{\overline{X} - \underline{X}}{k-1}.$$

Solving this equation gives

$$\overline{\pi}(k) = \frac{2k-1}{3k(k-1)}(\overline{X} - \underline{X}),$$

which is decreasing in k. The solution to the relaxed problem (C4) is then given by

$$N^* \begin{cases} = k & \text{if } \pi^R \in (\overline{\pi}(k+1), \overline{\pi}(k)) \\ \in \{k-1, k\} & \text{if } \pi^R = \overline{\pi}(k) \end{cases} \tag{C5}$$

Solving $\overline{\pi}(k) = \pi^R$ yields the solution (C3). From (C5), the solution to the relaxed problem is given by $N^* \in \{y-1, y\}$ if $y \in \mathbb{N}$ and $N^* = \lfloor y \rfloor$ otherwise.

Therefore, we are left to show that the above solution is indeed feasible under the constraint $(\overline{X} - \underline{X})/N \leq 2\pi^R$. For each π^R, the feasible number of intervals is greater than or equal to m, where m is the smallest natural number such that $(\overline{X} - \underline{X})/m \leq 2\pi^R$. The feasibility condition is satisfied if $N^* \geq m$. This is indeed the case because

$$\begin{aligned} N^* - m &\geq \frac{3\pi^R + 2(\overline{X} - \underline{X}) + \sqrt{9(\pi^R)^2 + 4(\overline{X} - \underline{X})^2}}{6\pi^R} - 1 - \frac{\overline{X} - \underline{X}}{2\pi^R} \\ &= \frac{-3\pi^R - (\overline{X} - \underline{X}) + \sqrt{9(\pi^R)^2 + 4(\overline{X} - \underline{X})^2}}{6\pi^R} \\ &\geq 0, \end{aligned}$$

where the last inequality is from

$$(9(\pi^R)^2 + 4(\overline{X} - \underline{X})^2) - (3\pi^R + (\overline{X} - \underline{X}))^2 = 3(\overline{X} - \underline{X})((\overline{X} - \underline{X}) - 2\pi^R) > 0.$$

□

Example 3 Revisited. In Example 3, we manually verified that $N^* = 5$ is optimal. Applying the above formula in this case ($X \sim \mathcal{U}[1,9]$ and $\pi^R = 1$), we have

$$y = \frac{19 + \sqrt{265}}{6} \approx 5.88,$$

so $N^* = \lfloor 5.88 \rfloor = 5$. ∎

Illustration of the Optimal Loss The auditor's optimal loss under the uniform partition is given by $\ell(\Delta^*)$, where $\ell(\Delta) = (\pi^R)^2 - \pi^R\Delta + \Delta^2/3$ and $\Delta^* = (\overline{X} - \underline{X})/N^*$. How much can the auditor gain by adopting this optimal vague communication strategy? To express the gain, write the optimal loss as a percentage of the default loss $(\pi^R)^2$:

$$\mathrm{RelLoss}(\Delta^*) := \frac{\ell(\Delta^*)}{(\pi^R)^2}.$$

When $\Delta^* = \Delta^{\text{ideal}} = 1.5\pi^R$, the loss is minimized at $\ell(\Delta^{\text{ideal}}) = 0.25(\pi^R)^2$. Moreover, for y defined in (C3), we have

$$\ell((\overline{X} - \underline{X})/y) \leq \ell((\overline{X} - \underline{X})/\lfloor y \rfloor) = \ell(\Delta^*),$$

so an upper bound on the optimal loss is given by

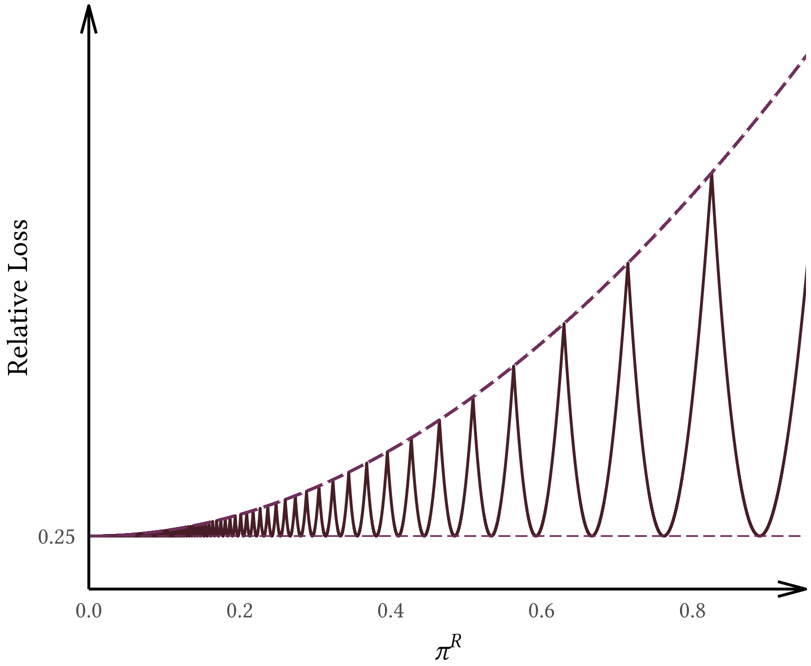
$$\ell((\overline{X} - \underline{X})/y) = (\pi^R)^2 + \frac{(\overline{X} - \underline{X})\left(2(\overline{X} - \underline{X}) - \sqrt{4(\overline{X} - \underline{X})^2 + 9(\pi^R)^2}\right)}{3}.$$

In sum, the optimal loss percentage satisfies the following bounds:

$$\mathrm{RelLoss}(\Delta^*) \in [0.25, \mathrm{RelLoss}((\overline{X} - \underline{X})/y)].$$

Figure C.2 illustrates the optimal loss percentage along with these bounds. When $\Delta^* = 1.5\pi^R$, the optimal loss percentage attains its lower bound of 0.25. As π^R increases from such a point, the loss initially rises, because the distortion from the ideal interval size grows. Once the loss reaches its upper bound, the optimal number of intervals, N^*, drops by one, and the loss percentage begins to fall.

Figure C.2: Optimal Loss as a Percentage of the Default Loss



Note: This figure plots the auditor's optimal loss under the uniform partition, expressed as a percentage of the default loss $(\pi^R)^2$. The solid line shows the optimal loss percentage as a function of π^R. The dashed curve depicts an upper bound on this percentage. The dashed horizontal line at 0.25 marks the lower bound of the optimal loss percentage.

C.3 Numerical Algorithm for the Optimal Partition Problem

In Proposition 2, I show that the optimal partition can be computed by solving a Bellman-type equation. Algorithm C.1 implements a dynamic program to compute it numerically. The algorithm iterates over a grid of right endpoints b and evaluates the loss function $L(b)$. For each b, it finds the optimal left endpoint $a = \lambda(b)$ as the solution to the recursive problem.

The algorithm works on a discretized version of $[\underline{X}, \overline{X}]$. For a given distribution, I compute the interval loss $\ell(a, b)$ by numerical integration and the no-experimentation constraint $\Gamma_E(b)$ by numerical root-finding. In Step 1, the algorithm constructs a grid of n right endpoints $\{b_i\}_{i=1}^n$. When $\underline{X} < -\pi^R$, we know that the leftmost interval is $[\underline{X}, -\pi^R]$ from Corollary A.2. In that case, I set the lower bound of the grid to $-\pi^R$, thereby focusing on the nontrivial portion of the partition. In Step 2, it initializes the overall loss function. The initialization can be arbitrary; given $L \leq (\pi^R)^2$, I set a schedule decreasing in $|b|$. Step 3 is the main part of the algorithm. In each iteration, for a fixed $L^{(\text{old})}$ from the previous iteration, the algorithm computes the overall loss

$$\mathbb{P}(X \geq b \mid X \leq a)\ell(a, b) + \mathbb{P}(X \leq b \mid X \leq a)L^{(\text{old})}(a).$$

Choosing the left endpoint a that minimizes this expression gives the optimal left endpoint and the updated loss $L^{(\text{new})}$. The iteration continues until $\|L^{(\text{new})} - L^{(\text{old})}\|_\infty$ falls below the tolerance. When the algorithm converges, we obtain the optimal left endpoints $\{a_i\}_{i=1}^n$ corresponding to the grid $\{b_i\}_{i=1}^n$.

Given these pairs $(a_i, b_i)_{i=1}^n$, Algorithm C.2 constructs the corresponding partition. Starting from the rightmost interval $[a_n, b_n]$, it traces back the sequence of intervals by taking the left endpoint of the current interval as the right endpoint of the next interval.

Algorithm C.1 Numerical Algorithm for the Optimal Partition Problem

```
input
    cumulative distribution function $F$; corresponding density $f$
    rejection payoff $\pi^R$
    grid max $X_{\max}$; grid size $n$
    tolerance $\varepsilon$; max iterations $T_{\max}$.

function $\ell(a, b)$
    return $\int_a^b (a + \pi^R - X)^2 / (F(b) - F(a))\, dF(x)$            ▷ Numerical integration

function $\Gamma_E(b)$                                                     ▷ The no-experimentation constraint
    function $h(a; b)$
        return $a + \pi^R - (F(b) - F(a))/f(a)$
    solve $h(a; b) = 0$ for $a$                                            ▷ Numerical root-finding
    return $a^*$ such that $h(a^*; b) = 0$

Step 1. BUILD RIGHT-ENDPOINT GRID WITH SAFEGUARD
    $\underline{b} \leftarrow \max\{\underline{X}, -\pi^R + 1 \times 10^{-3}\}$          ▷ Lower bound of the grid
    for $i = 1, \dots, n$ do
        $b_i \leftarrow \underline{b} + i \cdot (X_{\max} - \underline{b})/n$
    output $\{b_i\}_{i=1}^n$                                              ▷ Right-endpoint grid

Step 2. INITIALIZATION
    for $i = 1, \dots, n$ do
        $L^{(0)}(b_i) \leftarrow \dfrac{(\pi^R)^2}{1 + |b_i|}$                  ▷ Initialize loss (arbitrary)

Step 3. MAIN ITERATION LOOP
    $t \leftarrow 1;\ L^{(\mathrm{old})} \leftarrow L^{(0)};\ \mathrm{err} \leftarrow 1 \times 10^{10}$          ▷ Initialize loss and error
    while $t \le T_{\max}$ and err $> \varepsilon$ do
        for $i = 1, \dots, n$ do
            $a_{\min} \leftarrow \max\{b_i - 2\pi^R, \Gamma_E(b_i)\};\ a_{\max} \leftarrow b_i$
            $A(b_i) \leftarrow [a_{\min}, a_{\max}] \cap \{b_i\}_{i=1}^n$             ▷ Feasible left bounds (discretized)
            for all $a \in A(b_i)$ do
                $w(a, b_i) \leftarrow \mathbb{P}(X \le a \mid X \le b_i)$
                $\Lambda(a \mid b_i) \leftarrow \big(1 - w(a, b_i)\big)\, \ell(a, b_i) + w(a, b_i)\, L^{(\mathrm{old})}(a)$
            $a_i \in \arg\min_{a \in A(b_i)} \Lambda(a \mid b_i)$                    ▷ Optimal left bound
            $L^{(\mathrm{new})}(b_i) \leftarrow \Lambda(a_i \mid b_i)$
        err $\leftarrow \max_i |L^{(\mathrm{new})}(b_i) - L^{(\mathrm{old})}(b_i)|$             ▷ Update Error
        $L^{(\mathrm{old})} \leftarrow L^{(\mathrm{new})}$
        $t \leftarrow t + 1$
    output $\{(a_i, b_i,\ L^{(\mathrm{new})}(b_i))\}_{i=1}^n$
```

Algorithm C.2 Building Partition From Solution

```
input
    optimal partition from Algorithm C.1, ${(a_i, b_i)}_{i=1}^{n}$        ▷ Note $b_1 < b_2 < \cdots < b_n$

Step 1. INITIALIZATION
    $D_0 \leftarrow [a_n, b_n]$
    $c \leftarrow a_n$
    $k \leftarrow 1$

Step 2. BUILD PARTITION
    repeat
        $j \leftarrow \{j \mid b_j = c\}$        ▷ Take the previous left endpoint as the right endpoint
        $D_{-k} \leftarrow [a_j, b_j]$        ▷ Add to partition
        $c \leftarrow a_j$
        $k \leftarrow k + 1$        ▷ Go to next interval
    until $j = 1$
    $k_{\max} \leftarrow k$

    Ensure that the partition covers the entire support
    if $\underline{X} < \min D_{-k_{\max}}$ then $D_{-k_{\max}-1} \leftarrow [\underline{X}, \min D_{-k_{\max}}]$ else $D_{-k_{\max}-1} \leftarrow \emptyset$
    if $\max D_0 < \overline{X}$ then $D_1 \leftarrow [\max D_0, \overline{X}]$ else $D_1 \leftarrow \emptyset$

    output $\{D_{-k}\}_{k=-1}^{k_{\max}}$        ▷ Optimal partition
```

D The Manager with Private Information

In the main model, the manager does not have any private information. This is a simplifying assumption to focus on the auditor's communication strategies. In practice, managers often possess private information about the transaction that affects its accounting treatment. To incorporate this aspect, I now extend the model and assume that the manager has some private information. Let S be the manager's private information, which is a noisy signal about X. For simplicity, I assume that S is a "truth-or-noise" signal, i.e., S is either X or an independent noise from the same distribution as X. Denote by $\rho \in (0, 1]$ the precision of the signal—the probability that $S = X$. The manager observes S before choosing a report. The auditor does not observe S, but I continue to assume that she observes X.

The timeline of the game remains unchanged, except that the manager privately observes S before choosing a report. Unlike the baseline model, we will see that the auditor may sometimes prefer to induce reports that are not acceptable. Thus I analyze the case where the auditor's communication strategy encompasses both vague and precise communication: the message space is $\mathcal{M}(X) = \mathcal{M}^{\dagger}(X) \cup \mathcal{M}^{\overset{*}{**}}(X)$ (see (2) and (3)).

The upshot of the analysis is an intuitive result: the manager's private information makes it harder for the auditor to communicate her expertise without being exploited. The auditor does not observe the manager's private information, so she cannot tailor her communication to the manager's type. Consequently, the message may need to be overly informative to ensure that the manager's report is acceptable. Sometimes it is impossible to induce an acceptable report with vague communication. Even when it is possible, the auditor may prefer to provide more precise information and induce some unacceptable reports.

The analysis mimics the baseline model. First I solve for the manager's reporting problem, then I analyze the auditor's communication strategies.

D.1 The Reporting Problem with Private Information

When the auditor precisely reveals X, the ensuing subgame is exactly the same as the baseline model, because the auditor's precise communication renders the manager's private information irrelevant. In particular, the manager reports $X + \pi^R$, which makes the auditor indifferent.

Consider the manager's reporting problem when he learns that $X \in [a, b], a < b$ and that the private signal is S. Denote by ${}_aF_b(\cdot \mid S)$ the posterior cumulative distribution function of X conditional on $X \in [a, b]$ and the realization of S. Let $A(r \mid S)$ be the manager's posterior belief that the report r is accepted. The reporting problem with private information is as follows:

$$\max_{r \geq 0} rA(r \mid S). \tag{Report($a, b \mid S$)}$$

The manager updates belief with two pieces of information: the interval $[a, b]$ and the realization of S. The two signals are not independent, and the manager can use the information $X \in [a, b]$ to validate if the signal S is truthful or not. If $S \notin [a, b]$, the manager knows with certainty that S is pure noise. Conversely, if $S \in [a, b]$, the consistency of the two signals leads the manager to assign a higher likelihood that S is truthful. Specifically, in this case the manager's posterior belief that S is truthful is

$$w := \mathbb{P}(S = X \mid X \in [a, b], S) = \frac{\rho}{\rho + (1 - \rho)\mathbb{P}(X \in [a, b])} > \rho. \tag{D1}$$

Therefore, the manager's private information creates two types of the manager: the "informed type," who has some informative signal about X, and the "uninformed type," whose posterior belief is the same as the prior belief. The bilateral nature of the private information is now clear: the manager has only incomplete information about X, while the auditor does not know whether the manager is the informed type or the uninformed type.

Let $r(S, [a, b])$ be the manager's report when he learns that $X \in [a, b]$ and observes the signal S. It is helpful to define the reporting function for the informed and uninformed types separately:

$$r(S, D) = \begin{cases} r^I(S, D) & \text{if } S \in D, \\ r^U(D) & \text{if } S \notin D, \end{cases}$$

where $r^I(S, D)$ is the reporting function of the informed type and $r^U(D)$ of the uninformed.

The set of possible equilibrium outcomes is now more complicated than the baseline model. The manager's reporting problem now entails the signaling problem, where his report may reveal his type to the auditor. To set aside this issue, I restrict the analysis to the case where the two types of the manager make the same report. Therefore, the manager retains his private information in equilibrium, and the auditor faces uncertainty about the manager's type even after observing the report. One justification for this restriction is the following informal refinement argument: Suppose that the auditor is allowed to provide more information once she observes the manager's report. Then in a separating equilibrium, the auditor would like to be vague first and then provide more information once she learns that the manager is the better-informed type, who has a stronger incentive to gamble. In contrast, such a problem does not arise in the case of pooling equilibrium (or in the baseline model).

Under this restriction, the auditor's acceptance strategy remains unchanged: she accepts a report r if and only if $|r - X| \leq \pi^R$. Thus the uninformed manager's reporting problem also is the same as the baseline model. To consider the informed manager's reporting problem, first note that the posterior acceptance probability function is given by

$$A(r \mid S) = w\mathbf{1}_{\{|r-S|\leq\pi^R\}} + (1-w)A(r), \tag{D2}$$

where $A(r)$ is the acceptance probability function based on the prior belief. The manager places weight w on the event that his signal S is X, in which case he would choose the report $r = S + \pi^R$. If the signal is noise, the acceptance probability is the one based on the prior belief. When the manager is confident that his signal is precise (i.e., w is high), his report is closer to $S + \pi^R$.

Figure D.1 illustrates the acceptance probability function (D2) for the short-interval case ($b - a \leq 2\pi^R$). The thick solid line represents $A(r \mid S)$, which is a convex combination of the step function $\mathbf{1}_{\{|r-S|\leq\pi^R\}}$ and $A(r)$ (cf., Figure 2). Compared to $A(r)$, the manager's posterior acceptance probability $A(r \mid S)$ is strictly higher where $A(r)$ is decreasing. In that region, the manager's report will sometimes be rejected if the signal S turns out to be pure noise. Thus the possibility of the signal being precise strictly increases his perceived acceptance probability.

As in the main text, we can categorize the candidate solutions of Report($a, b \mid S$) for the short-interval case ($b - a \leq 2\pi^R$) into two options: the safe option and the risky option. The safe option, defined as the maximum report guaranteed to be accepted, is still $a + \pi^R$. The risky option is now defined as

$$r_S^+ = \arg\max_{r\geq 0}\ r\left[w\mathbf{1}_{r\leq S+\pi^R} + (1-w)A(r)\right]. \tag{D3}$$

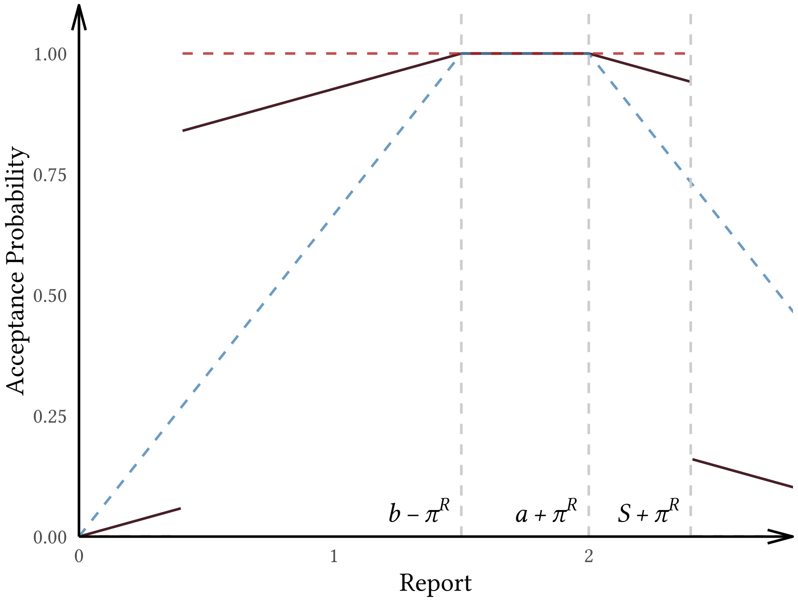
The report $r > a+\pi^R$ is more attractive than the no-private information case up until the acceptance threshold $S + \pi^R$, beyond which it becomes less attractive because it will be rejected for sure when the signal is precise.

D.2 The Auditor's Expected Loss with Private Information

When the auditor decides how much to communicate, she must consider the manager's private information. Let $r(S, D)$ be the manager's report when he observes S and the auditor communicates $D = [a, b]$. The auditor's expected loss when she observes X and communicates D is given by

$$\ell(D, X) := \mathbb{E}[\alpha(r, X)(r(S, D) - X)^2 + (1 - \alpha(r, X))(\pi^R)^2 \mid X].$$

Figure D.1: Acceptance Probability with Private Information



Note: The figure illustrates the acceptance probability $A(r \mid S)$ when $X \sim \mathcal{U}[1, 9]$ and $\pi^R = 1$. The interval is set as $[a, b] = [1, 2.5]$. The private signal realization is set as $S = 1.4$. The red horizontal dashed line represents the indicator function $\mathbf{1}_{|r-S|<\pi^R}$. The blue dashed curve represents the acceptance probability function without private information, $A(r)$.

The expectation is taken over the manager's private information S. Unlike the baseline model, the auditor does not know what the manager would report even if X and D are fixed. In particular, the auditor faces the informed manager, who knows that S is informative, and the uninformed manager, who ignores S. The informed manager is either the one who has a precise signal or the one who has a noisy signal. The lack of common knowledge is the key difference from the baseline model.

The auditor places weight on each of the three events (i.e., informed and precise signal, informed and noisy signal, and uninformed) based on her observation of X. From the auditor's perspective, with probability ρ, the manager is informed and has a precise signal, in which case the manager reports $r^I(X)$. With probability $(1 - \rho)\mathbb{P}(S \in [a, b] \mid S \neq X)$, the manager received a pure noise but does not realize this. In this case, the manager's report depends on the independent noise, which the auditor does not observe. Lastly, with probability $(1 - \rho)\mathbb{P}(S \notin [a, b] \mid S \neq X)$, the manager received a pure noise and knows that it is noise, in which case the manager reports

$r^U(D)$. Therefore, the auditor's expected loss function is given by

$$\begin{aligned}\ell(D, X) = \rho(\min\{|r^I(X) - X|, \pi^R\})^2 \\ + (1-\rho)\mathbb{P}(S \in [a, b] \mid S \neq X)\mathbb{E}[\min\{|r^I(S) - X|, \pi^R\}^2 \mid S \in [a, b], S \neq X] \\ + (1-\rho)\mathbb{P}(S \notin [a, b] \mid S \neq X) \min\{|r^U(D) - X|, \pi^R\}^2.\end{aligned}$$

D.3 The Communication Strategy with Private Information

No-Experimentation Constraint

In the baseline model without private information, the auditor optimally uses vague language to ensure that the manager's report is maximally acceptable (Lemma 2). To see how this result changes with private information, I first analyze the outcome in which the auditor ensures maximal acceptance. As in the baseline model, the informativeness of the message (i.e., the length of the interval) to ensure the manager chooses the safe option over the risky option reduces to the comparison of the two reports.

Proposition D.1. *Fix any $b > -\pi^R$. Given $b - a \leq 2\pi^R$, the manager chooses the safe option (i.e., $a + \pi^R > r_S^+$) if and only if*

$$a \in [\tilde{\Gamma}_G(b), b).$$

The lower bound $\tilde{\Gamma}_G(b) < b$ does not depend on the realization of S.

Proof. From (D3), it is clear that the manager with S such that $S + \pi^R \geq r$ has the stronger incentive to gamble. To prevent such a manager from gambling, the auditor must ensure that $\arg\max r(w+(1-w)A(r))$ is smaller than $a+\pi^R$. From the first order condition of the maximization problem, the condition reduces to

$$a + \pi^R \geq (1 + \hat{w})\frac{F(b) - F(a)}{f(a)}, \tag{D4}$$

where $\hat{w} := w/(1-w)$ is the posterior odds ratio. As in the proof of Lemma A.1, log-concavity implies the existence of a unique threshold of a, denoted by $\tilde{\Gamma}_G(b)$, that makes (D4) hold with equality. □

Notice that the no-gambling constraint does not depend on the realization of S. This is because the auditor does not observe S and thus must ensure that the manager chooses the safe option regardless of the realization of S.

With private information, a vague message that ensures an acceptable report may not exist: for a given $b > -\pi^R$, it is possible to have $\tilde{\Gamma}_G(b) > b$. To see why, suppose the manager's belief about the precision of his signal is fixed. In that case, providing more information to the manager discourages the manager from gambling, as the safe option becomes relatively more attractive In reality, the manager updates belief about the precision of his signal based on the message. If $[a, b]$ becomes smaller (more information), the manager who learns that $S \in [a, b]$ becomes more confident that his signal is precise. This strengthens the manager's incentive to gamble. Thus there are two opposing forces at play. If the latter force dominates, the auditor must provide even more information to reduce the manager's incentive to gamble, which in turn reinforces the manager's incentive to gamble. This feedback loop may lead to a situation where the only option to ensure an acceptable report is to reveal X precisely even when $X > -\pi^R$.

This outcome can be avoided when the manager's signal is sufficiently noisy.

Lemma D.1. *If ρ is low enough, then $\tilde{\Gamma}_G(b) < b$ for all $b > -\pi^R$.*

Proof. Observe that $\hat{w}$ is increasing in ρ and continuous in (a, ρ). When $\rho = 0$ (and thus $\hat{w} = 0$), inequality (D4) is equated by $a = \Gamma_G(b)$ (see the proof of Lemma A.1,). By continuity, for all $b > -\pi^R$, we can find a constant $\bar{\rho}_b > 0$ such that, given $\rho \in (0, \bar{\rho}_b)$, there is a threshold $\tilde{\Gamma}_G(b) < b$ that makes (D4) hold with equality. The proof of Lemma 3 establishes that $b - \Gamma_G(b)$ is increasing in b. Therefore $\bar{\rho}_b$ can be chosen to be increasing in b. Since $\underline{X} > -\pi^R$, we have $\inf_{b > \underline{X}} \bar{\rho}_b > 0$. □

The lemma says that if the manager's signal is not too precise, then there is always a vague message that ensures an acceptable report. In contrast, when the manager's private signal is precise enough, the manager's incentive is too strong so that it is impossible to induce an acceptable report. In this case, the auditor must provide precise information to prevent the manager from proposing an unacceptable report.

The Auditor's Communication Problem

Under the condition of Lemma D.1, the auditor can use vague communication to ensure that the manager's report is always acceptable. The auditor-optimal partition $\mathcal{D} = \{D_i\}_i$ with $D_i = [d_i, d_{i+1}]$, under the acceptance constraint is derived by (cf. OP′).

$$\begin{aligned} &\min_{\mathcal{D}} \sum \mathbb{P}(X \in D_i)\mathbb{E}[\ell(X, D_i) \mid X \in D_i], \\ &\text{s.t. } d_i \geq \tilde{\Gamma}(d_{i+1}) = \max\{\tilde{\Gamma}_G(d_{i+1}), d_{i+1} - 2\pi^R\}, \forall i \geq 1. \end{aligned} \tag{$\widetilde{\text{OP}}'$}$$

The main differences from the baseline model are the no-gambling threshold $\tilde{\Gamma}_G$ and the auditor's loss function $\ell(X, D_i)$. Under the acceptance constraint, $\alpha(r, X) \equiv 1$, so the auditor's expected loss D.2 reduces to

$$\begin{aligned}\ell(X, D_i) = \rho(r^I(X) - X)^2 \\ &+ (1-\rho)\mathbb{P}(S \in D_i \mid S \neq X)\mathbb{E}[(r^I(S) - X)^2 \mid S \in D_i, S \neq X] \\ &+ (1-\rho)\mathbb{P}(S \notin D_i \mid S \neq X)(r^U(D_i) - X)^2.\end{aligned}$$

The recursive formulation akin to Proposition 2 can be derived similarly to show that there is a unique solution to the problem $\widetilde{\mathrm{OP}}'$. Since the wishful off-path belief argument continues to hold, the optimal partition can be implemented in equilibrium.

However, ensuring maximal acceptance is generally not optimal with private information. This is because the auditor cannot tailor her communication to the manager's private information. Specifically, consider the auditor with realization X and the message $D = [a, b] \ni X$ that would be optimal if the auditor knows that the manager is uninformed (i.e., the baseline model). The informed manager has a stronger incentive to gamble. To ensure that he chooses the safe option, the auditor must provide more information—that is, choose a narrower interval $[a, b]$. But as $[a, b]$ becomes narrower, the probability that the manager becomes informed conditional on $X \in [a, b]$ declines.[3] In trying to prevent gambling by the informed manager, the auditor gives too much information to the uninformed manager—who is, in fact, much more likely.

The Uniform Distribution Case

To illustrate the above argument, I now specialize the analysis to the case where X follows the uniform distribution $\mathcal{U}[\underline{X}, \overline{X}]$ with $\underline{X} > -\pi^R$. Let $[a, b]$ be the auditor's message satisfying $b - a \leq 2\pi^R$. When $S \in [a, b]$, the risky option solves

$$\max_{r \geq 0} \; r\left[w\mathbf{1}_{r \leq S + \pi^R} + (1-w)\frac{b - (r - \pi^R)}{b - a}\right].$$

The manager with $S \geq r - \pi^R$ has the strongest incentive to inflate the report across realizations of $S \in [a, b]$. In this case, the risky option is given by $(b + \pi^R)/2 + \hat{w}(b - a)/2$. Therefore, the

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[3] From the auditor's perspective, the probability that the manager becomes informed given $X \in [a, b]$ is $\mathbb{P}(S \in [a, b] \mid X \in [a, b]) = \rho + (1-\rho)\mathbb{P}(S \in [a, b] \mid S \neq X)$.

no-gambling condition (D4) becomes

$$a + \pi^R \geq \frac{b + \pi^R}{2} + \hat{w}\frac{b - a}{2} \iff a \geq \frac{(1 + \hat{w})b - \pi^R}{2 + \hat{w}}. \tag{D5}$$

If $\hat{w}$ is fixed, then the right-hand side is greater than b, so the auditor can always provide more information to ensure that the manager chooses the safe option. However, the manager's belief about the precision of his signal, $\hat{w}$, is endogenous to the message $[a, b]$. From (D1), the posterior odds ratio is given by

$$\hat{w} = \frac{\rho}{1 - \rho}\frac{\overline{X} - \underline{X}}{b - a}.$$

Substituting this back into (D5), we obtain the no-gambling threshold:

$$\tilde{\Gamma}_G(b) = \frac{b - (1 - \rho)\pi^R - \rho\big(b - (\overline{X} - \underline{X})\big)}{2(1 - \rho)}.$$

Only when $\tilde{\Gamma}_G(b) < b$ does there exist a vague message that ensures an acceptable report, and this condition reduces to

$$\rho < \frac{b + \pi^R}{b + \pi^R + (\overline{X} - \underline{X})}. \tag{D6}$$

This upper bound corresponds to $\bar{\rho}_b$ in the proof of Lemma D.1. Since the right-hand side of (D6) is increasing in b, (D6) is satisfied for all possible messages if

$$\rho < \frac{\underline{X} + \pi^R}{\overline{X} + \pi^R}. \tag{D7}$$

This is the uniform upper bound on the signal precision claimed in Lemma D.1.

Suppose this condition holds. It is instructive to consider a simple partition that satisfies the acceptance constraint. Re-express the no-gambling constraint (D5) as a constraint on the right endpoint given a left endpoint a. Specifically, we solve (D5) for b to obtain

$$\tilde{\Psi}_E(a) = 2a + \pi^R - \frac{\rho}{1 - \rho}(\overline{X} - \underline{X}).$$

If $b \leq \tilde{\Psi}(a)$, then the manager chooses the safe option.

I construct a partition in $\mathcal{P}_A$ as follows. First, starting from the lower bound $\underline{X}$, I construct the first interval such that the no-gambling constraint binds: $D_0 = [\underline{X}, \tilde{\Psi}(\underline{X})]$. Then I iteratively construct the next interval $D_k = [\tilde{\Psi}^{(k)}(\underline{X}), \tilde{\Psi}^{(k+1)}(\underline{X})]$ such that the no-gambling constraint binds. I repeat this until the length of the next interval exceeds $2\pi^R$. At this point, the only relevant

Algorithm D.1 Constructing a Partition in $\mathcal{P}_A$

```
Step 1. NO-EXPERIMENTATION REGION
    $D_0 \leftarrow [\underline{X}, \tilde{\Psi}(\underline{X})]$
    $k \leftarrow 1$
    while $\tilde{\Psi}_E^{(k+1)}(\underline{X}) - \tilde{\Psi}_E^{(k)}(\underline{X}) \leq 2\pi^R$ do
        $D_k \leftarrow [\tilde{\Psi}_E^{(k)}(\underline{X}), \tilde{\Psi}_E^{(k+1)}(\underline{X})]$
    $\mathcal{D}_E \leftarrow \{D_1, \dots, D_k\}$
    $x_U \leftarrow \tilde{\Psi}_E^{(k+1)}(\underline{X})$.

Step 2. UNIFORM PARTITION
    $\mathcal{D}_U \leftarrow$ the optimal uniform partition for $\mathcal{U}[x_U, \overline{X}]$.

output $\mathcal{D}_E \cup \mathcal{D}_U$
```

constraint is $b - a \leq 2\pi^R$, so I construct the remaining intervals as the optimal uniform partition for $\mathcal{U}[x_U, \overline{X}]$, where x_U is the right endpoint of the last interval in the no-gambling region. Algorithm D.1 summarizes the construction.

The following example shows that the so-constructed partition is dominated by a partition that does not satisfy the acceptance constraint.

Example 2. Suppose that $X \sim \mathcal{U}[1, 9]$ and $\pi^R = 1$. Let the signal precision be $\rho = 0.15$, which satisfies (D7). The partition constructed by Algorithm D.1 is given by

$$\mathcal{D}_A = \{[1, 1.59], [1.59, 2.76], [2.76, 4.32], [4.32, 5.88], [5.88, 7.44], [7.44, 9]\}.$$

The first two intervals are in the no-gambling region, and the remaining intervals form the optimal uniform partition for $\mathcal{U}[2.76, 9]$. The auditor's expected loss under this partition is about 0.28.

Alternatively, suppose that the auditor ignores the manager's private information and uses the optimal uniform partition of size 5. Then the auditor's expected loss is about 0.27, which is lower than the loss under $\mathcal{D}_A$. Under this partition, rejection happens with positive probability, because the manager reports $r = S + \pi^R$ when S is small and $S \in D$. However, preventing such rejections is not worth it for the auditor, as she must provide too much information.

The dynamic programming approach under the acceptance constraint yields a partition similar to $\mathcal{D}_A$. The resulting loss is the same as that under $\mathcal{D}_A$ up to the fourth decimal place. This example illustrates that the auditor may prefer to induce unacceptable reports even when it is possible to ensure an acceptable report. ∎

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